Journal of Computational Physics 305 (2016) 1037–1064

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

An added-mass partition algorithm for fluid–structure interactions of compressible fluids and nonlinear solids

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ARTICLE INFO

Article history:
Received 15 July 2014
Received in revised form 3 August 2015
Accepted 27 October 2015
Available online 11 November 2015

Keywords:
Fluid–structure interaction
Compressible fluid flow
Hyperelastic solids
Riemann problems
Added-mass partitioned methods
Moving overlapping grids

ABSTRACT

We describe an added-mass partitioned (AMP) algorithm for solving fluid–structure interaction (FSI) problems involving inviscid compressible fluids interacting with nonlinear solids that undergo large rotations and displacements. The computational approach is a mixed Eulerian–Lagrangian scheme that makes use of deforming composite grids (DCG) to treat large changes in the geometry in an accurate, flexible, and robust manner. The current work extends the AMP algorithm developed in Banks et al. [1] for linearly elasticity to the case of nonlinear solids. To ensure stability for the case of *light* solids, the new AMP algorithm embeds an approximate solution of a nonlinear fluid–solid Riemann (FSR) problem into the interface treatment. The solution to the FSR problem is derived and shown to be of a similar form to that derived for linear solids: the state on the interface being fundamentally an impedance-weighted average of the fluid and solid states. Numerical simulations demonstrate that the AMP algorithm is stable even for light solids when added-mass effects are large. The accuracy and stability of the AMP scheme is verified by comparison to an exact solution using the method of analytical solutions and to a semi-analytical solution that is obtained for a rotating solid disk immersed in a fluid. The scheme is applied to the simulation of a planar shock impacting a light elliptical-shaped solid, and comparisons are made between solutions of the FSI problem for a neo-Hookean solid, a linearly elastic solid, and a rigid solid. The ability of the approach to handle large deformations is demonstrated for a problem of a high-speed flow past a light, thin, and flexible solid beam.

1. Introduction

Fluid–structure interaction (FSI) problems are important in many fields of engineering and applied science. These problems are often difficult to simulate in an efficient, stable and accurate manner. One important issue that arises in many FSI simulations is the treatment of large changes in geometry such as those arising when solid bodies or structures undergo large rotations and deformations. To address this challenge we use a mixed Eulerian–Lagrangian approach together with

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[1] This work was supported by contracts from the U.S. Department of Energy ASCR Applied Math Program.
[2] Research supported by a U.S. Presidential Early Career Award for Scientists and Engineers.
[3] Research supported by the National Science Foundation under Grant DMS-1519934.
[4] Research supported by the National Science Foundation under Grant DMS-1016188.

http://dx.doi.org/10.1016/j.jcp.2015.10.043
0021-9991/

deforming composite grids (DCG) that can flexibly and efficiently treat these large changes in geometry while retaining high quality grids [1]. A second important issue concerns the stability of the overall FSI algorithm and the numerical treatment of the interface between the fluid and solid domains. This is especially important for the case of partitioned solvers, which, unlike monolithic algorithms, make use of separate solvers for the fluid and solid domains. To address this issue we have developed a variety of *added-mass partitioned* (AMP) algorithms that embed analytically derived interface conditions into the numerical approximation at the interface as a means to obtain accurate and stable partitioned algorithms, even for *light* solids when added-mass effects are large. The form of the derived interface conditions depends on the regime of the FSI problem. For the case of inviscid compressible fluids coupled to rigid solids [2], for example, the derived AMP interface conditions incorporate added-mass tensors into the rigid-body equations that stabilize the AMP scheme when the mass of the body is small, or even zero. For cases involving deformable solids, on the other hand, the derived AMP interface conditions take the form of mixed (Robin) type conditions involving the velocity and stress; the coefficients in the mixed interface conditions account for the added mass effects. This approach was first used in [1,3] for the case of inviscid compressible fluids coupled to linearly elastic solids, and later extended to FSI regimes involving *incompressible* fluids coupled to either compressible elastic bulk solids [4] or elastic structural shells [5,6]. A principal aim of the present paper is to extend the analysis in [1] and develop an AMP algorithm for inviscid compressible fluids coupled to nonlinear hyperelastic solids. This new AMP algorithm is applicable to a large class of FSI problems, such as those involving large solid deformations and rotations.

We consider a mixed Eulerian–Lagrangian formulation of the FSI problem. The equations governing the fluid are formulated in *physical (Eulerian) space* with evolving boundaries, and are given by the equations of gas dynamics with an ideal equation of state. For the solid, we formulate the governing equations in a static *reference (Lagrangian) space*, and write these equations as a first-order system for the components of displacement, velocity and nominal stress. A constitutive law relating the stress tensor to the deformation gradient tensor completes the system of equations for the solid. We consider the class of hyperelastic solids which implies a nonlinear constitutive law in general. The mathematical formulation of the FSI problem is closed by specifying initial conditions, by imposing matching conditions for velocity and stress on the deforming fluid–solid interface, and by assigning suitable boundary conditions on the remaining portions of the fluid and solid domains.

The equations governing the fluid and solid are both systems of nonlinear hyperbolic partial differential equations (assuming certain conditions on the deformation in the solid depending on the choice of constitutive law). The equations are solved numerically on DCGs. For the fluid, we use a general arbitrary Lagrangian–Eulerian (ALE) formulation of the equations on moving grids, and solve the equations using a second-order extension of Godunov's method with an approximate Roe Riemann solver. This Godunov scheme is described in [7] for static boundaries, and extended in [8] for moving boundaries. The numerical treatment of the equations in the fluid domain are well described elsewhere and we provide only a very brief description in this paper for completeness. For the solid, the equations are mapped to a static computational space and solved on overlapping grids using a new second-order accurate, characteristics-based upwind method, which extends the approach in [9] to nonlinear elasticity. The characteristics of the system of equations for the solid with nonzero wave speed correspond to pressure and shear waves. There are also characteristics with zero wave speed corresponding to the tangential components of stress and the components of displacement. The upwind scheme for the solid adds no dissipation to the characteristic variables associated with these zero wave speeds, and this can lead to numerical instabilities for certain problems. To suppress these instabilities, we add small high-order dissipation terms to the components of the solution corresponding to the zero wave speeds.

There are two other (related) issues that arise when solving the first-order system for the evolution of the solid. The first issue concerns the compatibility between the stress and strain. For the first-order system, we evolve the time derivative of the stress–strain relation, $\bar{P} = \bar{\mathcal{P}}(\bar{F})$, where $\bar{P}$ and $\bar{F}$ denote the nominal stress and deformation gradient tensors, respectively, and thus the numerical solution can drift from satisfying the stress–strain relation. This drift can be suppressed by adding a stress–strain relaxation term (penalty term) to the evolution equation for $\bar{P}$ following the approach used in [1]. The second issue is that although the momentum equation is in conservation form, the evolution equation for $\bar{P}$ is not. This is a potential issue when solving problems with discontinuous jumps in the velocity that can occur, for example, when a fluid shock impacts the solid. In this case we have found that use of the stress–strain relaxation term can ameliorate issues related to the non-conservative form. This derives from the fact that satisfaction of the stress–strain relation together with the conservative treatment of the momentum equation would lead to the correct weak solution for convergent numerical schemes. We note that there are alternative conservative formulations of the first-order system, such as the one considered by Miller and Colella [10], for example.

The coupling of the numerical solutions for the fluid and solid is performed at the fluid–solid interface using an *interface projection scheme*. The scheme embeds an approximate solution of a nonlinear fluid–solid Riemann (FSR) problem into the numerical treatment of the interface. The solution to the FSR problem leads to formulas for the projected states of velocity and stress that are impedance-weighted averages of the predicted fluid and solid states adjacent to the interface. These formulas, which we refer to as the *full projection*, are extensions of the ones derived in [1] for a linearly-elastic solid. In addition to the full projection, we also derive a *simplified projection* which is easier to implement and computationally less costly. Numerical results indicate that the simplified projection provides nearly identical results to those given by the full projection for the problems considered. The new AMP scheme based on the full or simplified projection remains stable even for light solids when added-mass effects are large.