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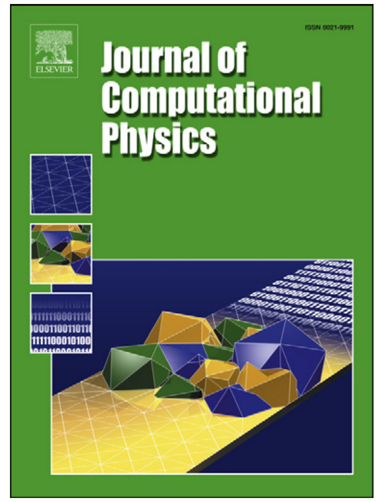

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A. Adimurthi, Aekta Aggarwal, G.D. Veerappa Gowda

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Godunov-type Numerical Methods for a Model of Granular Flow

A. Adimurthi[1] Aekta Aggarwal[1,2] G. D. Veerappa Gowda[1]

This paper is dedicated to Professor Jérôme Jaffré on the occasion of his 65^{th} *birthday.*

Abstract

We propose and analyze finite volume Godunov type methods based on discontinuous flux for a 2×2 system of non-linear partial differential equations proposed by Hadeler and Kuttler to model the dynamics of growing sandpiles generated by a vertical source on a flat bounded rectangular table. The scheme is made well-balanced by modifying the flux function locally by including source term as a part of the convection term. Its extension to multi–dimensions is not straightforward for which an approach has been introduced here based on *Transport Rays*. This approach is compared with another approach for inclusion of source term which uses the idea of inverting the divergence operator relying on the Curl–free component of the Helmholtz decomposition of the source term. Numerical experiments are presented to illustrate the efficiency of the proposed scheme for both unsteady and steady state calculations and to make comparisons with the previously studied finite difference and semi-Lagrangian approaches by Falcone and Finzi Vita in [17, 19].

Keywords: Balance Laws, Discontinuous Flux, Granular Matter, Well–Balanced Schemes, Finite volume, Finite Difference, Transport Rays

2010 MSC: 35L65, 65M12

1 Introduction

In the last decade several papers have been devoted to the study of the dynamics of granular matter since a complete and realistic description of many phenomena in this field is not completely available. Many mathematical models have been derived using different techniques coming from kinetic, differential equations or cellular automata theories, see [19] and the references therein. This field of research, which is of strong relevance in the applications, has also been the source of many new and challenging problems in the theory of partial differential equations (see, e.g., [11, 12, 13, 14, 16, 35]) and also in the numerical approximations of the proposed models (see, e.g., [17, 18]).

We are concerned with the evolution of a sandpile created by pouring dry sand grains, all of same size(to neglect phenomena of segregation or formation of patterns), on a flat bounded table under the effect of an external vertical source, neglecting the external effects such as wind or stress field in the bulk of the medium. The flat bounded table is represented by an open bounded domain $\Omega \subset \mathbb{R}^2$ and the time-independent non-negative vertical source by $f \in L^1(\Omega)$. There are two main models based on partial differential equations to describe this phenomenon. The first one is the variational model of Prigozhin [35], where the surface flow of sand is supposed to exist only at critical slope α, that is the maximal admissible slope for any stationary configuration of sand. No matter can accumulate on slopes steeper than α and no pouring over the parts of the pile

[1]TIFR Centre for Applicable Mathematics, Bangalore 560065, India (aditi@math.tifrbng.res.in, gowda@math.tifrbng.res.in)

[2]The Indian Institute of Management Indore, Prabandh Shikhar, Rau Pithampur Road, Indore, Madhya Pradesh 453556, India (aektaaggarwal@iimidr.ac.in)