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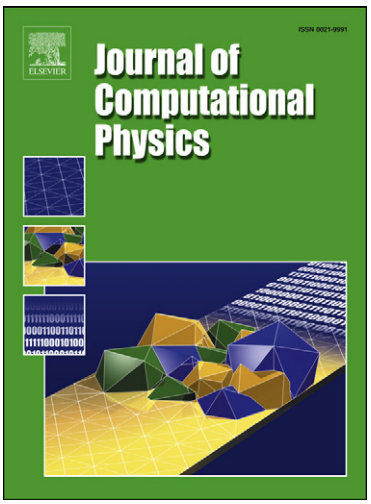

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Fast and Stable Explicit Operator Splitting Methods for Phase-Field Models

Yuanzhen Cheng*, Alexander Kurganov†, Zhuolin Qu‡ and Tao Tang§

Abstract

Numerical simulations of phase-field models require long time computations and therefore it is necessary to develop efficient and highly accurate numerical methods. In this paper, we propose fast and stable explicit operator splitting methods for both one- and two-dimensional nonlinear diffusion equations for thin film epitaxy with slope selection and the Cahn-Hilliard equation. The equations are split into nonlinear and linear parts. The nonlinear part is solved using a method of lines together with an efficient large stability domain explicit ODE solver. The linear part is solved by a pseudo-spectral method, which is based on the exact solution and thus has no stability restriction on the time-step size. We demonstrate the performance of the proposed methods on a number of one- and two-dimensional numerical examples, where different stages of coarsening such as the initial preparation, alternating rapid structural transition and slow motion can be clearly observed.

Key words: Phase-field models, molecular beam epitaxy equation, Cahn-Hilliard equation, operator splitting methods, semi-discrete finite-difference schemes, large stability domain explicit Runge-Kutta methods, pseudo-spectral methods, adaptive time-stepping

AMS subject classification: 65M99, 65M20, 65M70, 35K25, 76A20, 35Q99, 65M12

1 Introduction

Phase-field models have been recently introduced to describe interfacial phenomena. They were originally derived for the microstructure evolution and phase transition, but have been recently extended to many other physical phenomena, such as solid-solid transitions, growth of cancerous tumors, phase separation of block copolymers, dewetting and rupture of thin liquid films and infiltration of water into porous medium.

Two of these phase-field models have attracted much attention: the molecular beam epitaxy (MBE) equation with slope selection

$$u_t = -\delta\Delta^2 u + \nabla\cdot f(\nabla u), \quad (x,y)\in\Omega\subset\mathbb{R}^2,\ t\in(0,T], \tag{1.1}$$

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