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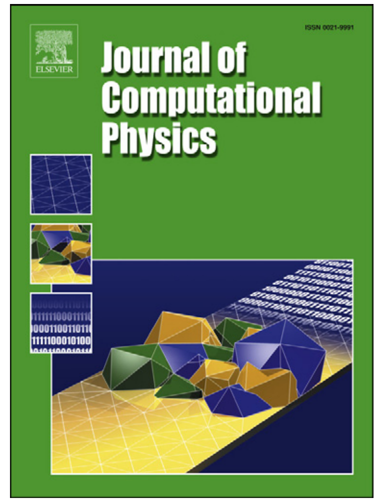

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Implicit, semi-analytical solution of the generalized Riemann problem for stiff hyperbolic balance laws

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Abstract

We present a semi-analytical, implicit solution to the generalized Riemann problem (GRP) for non-linear systems of hyperbolic balance laws with stiff source terms. The solution method is based on an implicit, time Taylor series expansion and the Cauchy-Kowalewskaya procedure, along with the solution of a sequence of classical Riemann problems. Our new GRP solver is then used to construct *locally implicit ADER methods* of arbitrary accuracy in space and time for solving the general initial-boundary value problem for non-linear systems of hyperbolic balance laws with stiff source terms. Analysis of the method for model problems is carried out and empirical convergence rate studies for suitable tests problems are performed, confirming the theoretically expected high order of accuracy.

Keywords:
Hyperbolic balance laws, stiff source terms, generalized Riemann problem, Cauchy-Kowalewskaya procedure, high-order ADER schemes

1. Introduction

This paper is motivated by the fully discrete one-step ADER approach to construct numerical schemes of arbitrarily high order of accuracy in space and time for solving hyperbolic equations. ADER (Arbitrary Accuracy DERivative Riemann problem method) was first put forward by Toro et al. [41], in the finite volume framework, for solving linear hyperbolic equations in one and multiple space dimensions on Cartesian meshes; see also Schwartzkopff, Munz and Toro [35]. The extension of finite volume ADER schemes (ADER-FV) to non-linear equations, due to Titarev and Toro [38], is based on a semi-analytical, explicit solution of the Generalized Riemann problem put forward by Toro and Titarev [43]. Since then, ADER has also been extended to the discontinuous Galerkin finite element framework by Dumbser [17], giving rise to ADER-DG schemes.

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