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A new multi-scale structure finding algorithm to identify cosmological structure

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ABSTRACT

We introduce a new structure finding algorithm that self-consistently parses large scale cosmological structure into clusters, filaments and voids. This structure finding algorithm probes the structure at multiple scales and classifies the appropriate regions with the most probable structure type and size. We show that it can identify the baryon fraction of intercluster medium and cosmological voids.

1. Introduction

Numerous large galaxy redshift surveys have revealed filaments and walls connecting clusters together with vast voids in between. This large scale structure (LSS) range in size from several to hundreds of megaparsecs. Location within the LSS affects the gas density, temperature, and chemical enrichment history, the galaxy merger and harassment history and the orientation of galaxies [1] and references therein. One of the primary motives in developing an algorithm that self-consistently identifies groups/clusters (henceforth jointly referred to as clusters), filaments and voids is to probe the effect of environment on galaxy evolution.

The challenge is to find an optimum algorithm that can successfully identify clusters, filaments and voids in large observational and numerical data sets. Many different methods [2,3] and references therein have been proposed for identifying structures and their associated galaxies within large galaxy redshift surveys and cosmological simulations. Most algorithms follow one particular type of structure. Moreover, there are significant ambiguities in the definitions of what exactly constitutes a filament or void (clusters are well defined), both observationally and in simulations [4]. Hence, a comparison of structures found by different algorithms remains a challenging task [4].

Only a few algorithms [5,6] follow multiple types of structure. We present a new structure finding algorithm adapted to solve this problem. It is similar to those presented by Refs. [6,5,7]. It utilizes the density curvature information provided

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by the eigenvalues of the Hessian matrix to find structures. However, unlike Refs. [5,7], the structure finder presented here provides a self-consistent way for dealing with clusters, filaments *and* voids.

This new algorithm also has the advantage of being computationally more efficient than the structure finders of Refs. [5,6]. We use the density obtained directly from the simulation output, without the need to compute either a Delaunay Tessellation Field Estimator [5] or the gravitational potential and particle trajectories [6].

2. Methods

The structure finding algorithm described here uses 2nd order curvature information from the density distribution to parse the large scale structure. It is based on a vasculature segmentation algorithm [8] used in medical imaging. The algorithm uses the local curvature information to filter [9,8] the filaments from other structures. To get this curvature information one can Taylor expand the density, $\rho(\mathbf{x},\sigma)$, about the local position $\mathbf{x_0}$ at some characteristic scale, σ. The resulting expansion is

$$\rho(\mathbf{x_0}+\delta\mathbf{x},\sigma) \approx \rho(\mathbf{x_0},\sigma) + \nabla\rho(\mathbf{x_0},\sigma)_i \delta\mathbf{x}^i + \frac{1}{2}H(\mathbf{x_0},\sigma)_{ij}\delta\mathbf{x}^i\delta\mathbf{x}^j + \ldots, \tag{1}$$

where $\nabla\rho(\mathbf{x_0},\sigma)_i$ is the gradient of the density, and $H(\mathbf{x_0},\sigma)_{ij}$ is the 3×3 Hessian matrix comprised of the second derivatives of the density. To calculate the spatial derivatives, the density field is convolved [9,10] with a kernel composed of the derivatives of a Gaussian function along the desired dimension. Thus,

$$H(\mathbf{x_0},\sigma)_{ij} = \int \nabla\rho(\mathbf{x_0},\sigma)_i * \frac{\partial G(\mathbf{x}-\mathbf{x_0},\sigma)}{\partial x_j} dx_j\ , \tag{2}$$

where

$$\nabla\rho(\mathbf{x_0},\sigma)_i = \int \rho(\mathbf{x},\sigma) * \frac{\partial G(\mathbf{x}-\mathbf{x_0},\sigma)}{\partial x_i} dx_i \tag{3}$$

and

$$G(\mathbf{x}-\mathbf{x_0},\sigma) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\mathbf{x}-\mathbf{x_0})^2}{2\sigma^2}}. \tag{4}$$

The characteristic Gaussian scale, σ, is iterated through a range of values to effectively probe structures of different sizes. The algorithm returns the maximum structure measure and the corresponding scale (σ) for each cell or voxel. Thus larger structures are probed best by wider Gaussian kernels and smaller structures by smaller Gaussian kernels.

The resulting eigenvectors yield three orthogonal axes with the eigenvalues giving the relative spatial curvature along each axis [9]. These eigenvalues are ordered by their relative magnitudes as

$$|\lambda_1| \le |\lambda_2| \le |\lambda_3|. \tag{5}$$

The relative ratios of the eigenvalues determines the structural information. For example, if $|\lambda_1| << |\lambda_2|$ and $|\lambda_2| \approx |\lambda_3|$, then, one dimension has a relatively small curvature, while two dimensions have much higher curvature, which indicates a filament.

We expand this algorithm to find clusters and voids. The relative ratios of the eigenvalues are used to generate a unique structure measure for each type of structure. Following the logic from Refs. [9,8], how cluster-like or filament-like a region of space is can be quantified by the structure measures

$$V_c = (1 - e^{-2|\lambda_1/\lambda_3|^2})(1 - e^{-2|F_{norm}|^2}) \tag{6}$$

and

$$V_f = (1 - e^{-2|\lambda_1/\lambda_2|^2})(1 - e^{-2|F_{norm}|^2})\ , \tag{7}$$

respectively, where F_{norm} is the Frobenius norm,

$$F_{norm} = ({\lambda_1}^2 + {\lambda_2}^2 + {\lambda_3}^2)^{\frac{1}{2}}. \tag{8}$$

The Frobenius norm term acts to wash out structure signal by suppressing small eigenvalue fluctuations. Building upon this logic, we introduced a new structure measure that is useful for voids

$$V_v = (1 - e^{-2|\lambda_1/\lambda_3|^2})(e^{-2|F_{norm}|^2}). \tag{9}$$

Here, the role of the Frobenius norm term has changed. Instead of decreasing the signal from small eigenvalue fluctuations, it enhances the signal. The reasoning is that regions of low density will be dominated by the small eigenvalue fluctuations not related to any physical structure. This provides a self-consistent method with which to investigate structures observed in the Universe.