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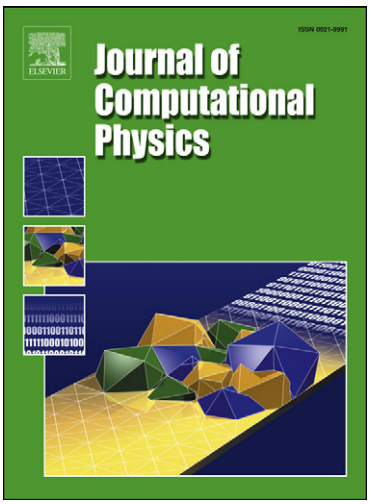

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Fractional Sturm-Liouville boundary value problems in unbounded domains: Theory and applications

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Abstract

Recently, Zayernouri and Karniadakis in [M. Zayernouri, G. E. Karniadakis, *Fractional Sturm–Liouville eigen–problems: Theory and numerical approximation*, Journal of Computational Physics, **252** (2013) 495-517] investigated two classes of fractional Sturm-Liouville eigenvalue problems on compact interval $[a, b]$ in more details. They were the first authors who not only obtained some explicit forms for the eigensolutions of these problems but also derived some useful spectral properties of the obtained eigensolutions. Until now, to the best of our knowledge, fractional Sturm-Liouville eigenvalue problems on non-compact interval, such as $[0, +\infty)$ are not analyzed. So, our aim in this paper is to study these problems in details. To do so, we study at first fractional Sturm-Liouville operators (FSLOs) of the confluent hypergeometric differential equations of the first kind and then two special cases of FSLOs; FSLOs-1 and FSLOs-2 are considered. After this, we obtain the analytical eigenfunctions for the cases and investigate the spectral properties of eigenfunctions and their corresponding eigenvalues. Also, we derive two fractional types of the associated Laguerre differential equations. Due to the non-polynomial nature of the eigenfunctions obtained from the two fractional associated Laguerre differential equations, they are defined as generalized associated Laguerre functions of the first and second kinds, GALFs-1 and GALFs-2. Furthermore, we prove that these fractional Sturm-Liouville operators are self-adjoint and the obtained eigenvalues are all real, the corresponding eigenfunctions are orthogonal with respect to the weight function associated to FSLOs-1 and FSLOs-2 and form two sets of non polynomial bases. At the end, two new quadrature rules and $\mathbf{L}^2$-orthogonal projections with respect and based on GALFs-1 and GALFs-2 are introduced. The upper bounds of the truncation errors of these new orthogonal projections according to some prescribed norm are proved and then verified numerically with some text examples. Finally, some fractional differential equations are provided and analyzed numerically.

Keywords: Liouville fractional derivatives and integrals, Confluent hypergeometric function, Associated Laguerre polynomials, Fractional Sturm-Liouville problems, Self-adjoint operators, Eigenvalues and eigenfunctions, Modified Gauss-Laguerre quadrature rules, Orthogonal projections, Truncation errors.
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