Journal of Computational Physics 299 (2015) 597–612

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Artificial boundary layers in discontinuous Galerkin solutions to shallow water equations in channels

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ARTICLE INFO

Article history:
Received 17 July 2013
Received in revised form 2 April 2015
Accepted 10 July 2015
Available online 14 July 2015

Keywords:
Discontinuous Galerkin
Finite elements
Shallow water equations
Open channel
Numerical fluxes
No-normal flow boundary condition
Iso- and super-parametric finite elements

ABSTRACT

In this work, we consider the application of Discontinuous Galerkin (DG) solutions to open channel flow problems, governed by two-dimensional shallow water equations (SWE), with solid curved wall boundaries on which the no-normal flow boundary conditions are prescribed. A commonly used approach consists of straightforwardly imposing the no-normal flow condition on the linear approximation of curved walls. Numerical solutions indicate clearly that this approach could lead to unfavorable results and that a proper treatment of the no-normal flow condition on curved walls is crucial for an accurate DG solution to the SWE. In the test case used, errors introduced through the commonly used approach result in artificial boundary layers of one-grid-size thickness in the velocity field and a corresponding over-prediction of the surface elevation in the upstream direction. These significant inaccuracies, which render the coarse mesh solution unreliable, appear in all DG schemes employed including those using linear, quadratic, and cubic DG polynomials. The issue can be alleviated by either using an approach accounting for errors introduced by the geometric approximation or an approach that accurately represents the geometry.

1. Introduction

The shallow water equations (SWE) serve as an excellent model for incompressible flow with horizontal scales much larger than depth. The SWE are used extensively in modeling many environmental flows, such as tides, hurricane induced coastal flooding, open channel and riverine flow. Simulation of these problems often involves large, geometrically complicated domains and integration over a long period of time. Numerical methods to accurately solve the SWE must be able to propagate long waves and accurately simulate convective processes. Successful continuous Galerkin (CG) finite element solutions to the SWE include, but are not limited to, those devised in [1–4]. Discontinuous Galerkin (DG) finite element methods (see [5–7] and references therein for reviews and detailed accounts of DG methods), which excel in the solution of propagation- and convection-dominated problems, have emerged as a powerful alternative for solving the SWE [8–15]. Conceptually similar to finite volume (FV) methods, DG methods inherently posses the property of being conservative on

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http://dx.doi.org/10.1016/j.jcp.2015.07.015

the elemental level, a desirable property when coupling flow and transport equations. Unlike FV methods, high-order DG schemes on unstructured meshes can be constructed in a straightforward manner. Since they employ a piecewise discontinuous approximation, DG methods are able to accommodate non-conforming meshes and allow the use of polynomial approximations of arbitrary order in each element, thus making them naturally well-suited for an *hp*-adaptive discretization. In addition, the parallel implementation of DG schemes is highly scalable when used in conjunction with explicit time integration schemes [10].

While DG methods have numerous favorable properties, one major drawback of DG solutions in comparison to CG solutions on a given mesh is the larger number of degrees of freedom, which directly implies greater computational costs. The performance study of DG and CG methods for the SWE in [16] demonstrates that, for linear elements on identical meshes, the cost per time step of the DG solution [8,9] is approximately four to five times higher than that of the CG solution [2] (the latter solves the generalized wave continuity equation, a reformulated form of the SWE). Such a higher cost is not as alarming as it seems as the subsequent study [10] demonstrates that the DG method has comparable or higher efficiency in terms of obtaining a specified error level for a given computational cost and in terms of scalability on parallel machines. Most SWE solvers are first and second order accurate methods that are based on cell-averaged FV and linear finite elements. Indeed, for problems with smooth solutions, as demonstrated in [17,18], DG solutions offer a significant computational cost-per-accuracy when using high-order elements, i.e. elements with polynomial interpolants of degree p greater than unity.

Developments made over the years, described in a number of papers [8–10,12,19], enable Dawson et al. [13] to apply the linear-element DG methods to a realistic modeling of hurricane-induced coastal and inland flooding. In [13], the results from linear-element DG methods are validated against the observation data and compared with the results from ADvanced Circulation (ADCIRC) code [2], a CG-based SWE solver used extensively in such applications. Solutions, computed with an identical high-resolution mesh and physical parameter values, from these two methods agree well in most of the domain; however, significant disagreement in the results is seen in inland areas, especially in meandering channels. In the channels, the surge level of the ADCIRC solution is in good agreement with the observation data. However, the surge level in the DG solution is damped compared to that of the ADCIRC solution and attenuates at a faster rate in the upstream direction. To a certain degree, this indicates that the DG solution is more diffusive in channels and hinders DG methods from becoming a viable tool in storm-surge applications.

In this work, motivated in part by an attempt to resolve the issue mentioned above, we investigate the effect of curved-wall boundary treatments in DG solutions for SWE to open channel flow. As widely employed in CG calculations, DG calculations simply replace channel curved walls with a linear approximation (see for example [20,21]) and apply the no-normal flow condition on each straight segment in a straightforward manner. In gas dynamics, Bassi and Rebay [22] demonstrate that DG solutions are highly sensitive to the accuracy of the representation of a solid curved wall, a boundary on which the no-normal flow condition is prescribed. Numerical results shown therein (also see [23]) demonstrate that the DG methods under p-refinement fail to yield a numerical solution that converges to the true solution when imposing the no-normal flow (or slip) condition on the linear approximation of the geometry, i.e. on a set of straight segments. Errors introduced by the geometric approximation appear to have a strong effect on the solution away from the boundary. Bassi and Rebay [22] show that this issue can be resolved by approximating the geometry using a polynomial of degree that is at least equal to the degree of the DG polynomial but not less than two, i.e. using at least iso-parametric elements for $p > 1$ and super-parametric elements for $p = 1$ for boundary-mesh elements. As shall be seen in detail in Section 5, simply prescribing the no-normal flow condition on the linear approximation of the solid curved wall of the channel leads to the presence of resolution-dependent artificial boundary layers and an over-prediction of the surface elevation on the upstream side. In this work, in addition to considering the curvilinear iso- and/or super-parametric elements, we employ a so-called curvature-boundary-condition approach, proposed originally for the Euler equations in [23], for the treatment of the no-normal flow condition on solid curved walls. Such an approach adjusts a component enforcing the boundary condition in a DG formulation so that the physical no-normal flow conditions are better approximated on the straight-sided-element mesh.

The remainder of the paper is organized as follows. In Section 2, we provide a description of the two-dimensional SWE. A DG method for SWE described in [8,9] is briefly summarized in Section 3 (also, we briefly discuss considerations to achieve a so-called well-balanced property in high-order DG schemes in this section). Section 4 contains the detailed account of the two different approaches for treating the no-normal flow condition on a solid wall. In this study, a converging/diverging channel problem is used as a test problem and is described in Section 5.1. Section 5.2 presents results from the study on the flow problem. Conclusions are drawn in Section 6.

2. Governing equations

By assuming a hydrostatic pressure distribution and a uniform velocity profile in the vertical direction, flow in a channel can be modeled by two-dimensional shallow water equations (SWE), also known as the St. Venant equation. The SWE consist of the depth-averaged continuity equation and x- and y-momentum equations written here in a conservative form as:

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot \mathbf{F}(\mathbf{q}) = \mathbf{s}(\mathbf{q}, \boldsymbol{x}, t), \quad (\boldsymbol{x}, t) \in \Omega \times [0, \infty) \tag{1}$$