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Isogeometric analysis based on extended Loop's subdivision

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ABSTRACT

In this paper, we present a new approach of isogeometric analysis (IGA) based on the extended Loop's subdivision scheme. This strategy allows us to integrate geometric modeling and physical simulation. The geometries can be open and with holes. Our proposed method performs geometric modeling via the extended Loop's subdivision which allows arbitrary topological structure, treats concave/convex vertices, and has at least C^1-continuity everywhere. It is capable of handling domains with arbitrary shaped boundary represented by piecewise cubic B-spline curves. We apply an efficient integration technique to the domain elements with a fast evaluation technique for closed Loop's subdivision surfaces. As an example, the Poisson equation is solved on three planar domains. We develop the approximate estimation of finite element in the limit function space of the extended Loop's subdivision. A detailed study on the convergence character is given with the comparison to the classical finite element analysis (FEA) with linear elements. Numerical experiments are consistent with our theoretical results. It shows that compared with the FEA with linear elements, the IGA scheme based on extended Loop's subdivision converges faster and behaves more robustly with respect to the mesh quality.

1. Introduction

The finite element method is a vital technique in solving partial differential equations (PDEs), which has a more than fifty years history of development and accumulated a strong mathematical foundation. The central idea is using piecewise polynomial functions over a finite-dimensional space to approximate the solution of PDEs. Classical finite element analysis technology adopts some low order finite elements as approximation and analysis tools for geometry. Therefore, for Computer Aided Design (CAD) objects, there exists a mesh generation process in order to get their computational domains, which is very time-consuming and brings undesirable discretization errors.

IGA was originally introduced by Hughes et al. [20] which uses volumetric Non-uniform Rational B-splines (NURBS) [12,22] or T-splines [21,23–25] to replace traditional finite elements, therefore it is able to improve the efficiency, quality and accuracy during the analysis procedure. IGA integrates CAD and FEA by adopting the uniform representation such as NURBS to describe the geometry and perform the numerical analysis. It avoids the difficulty of mesh generation. Moreover, we

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can use h-refinement by knot insertion, and p-refinement by order elevation to improve the simulation accuracy without changing the geometry. NURBS require structured initial meshes. To support more flexible geometry representation from design, IGA has incorporated T-splines into analysis, which possess T-joints and supports local refinement. The locally refined B-splines, denoted LR B-splines, is recently proposed in [9] as an implementation of T-splines.

Surface subdivision is a powerful technique in surface design which can efficiently generate smooth surfaces from arbitrary initial meshes through a simple refinement process. It is capable of constructing surfaces with no limitation on the topology of the control meshes, and recovering sharp features such as creases and corners. Subdivision surfaces and functions defined on them have played a key role in computer graphics and numerical analysis. A class of piecewise smooth surface representations in [7] were introduced based on subdivision to reconstruct smooth surface from scattered data. Thin-shell finite element analysis [4] was used for describing both the geometry and associated displacement fields. The limit function representation of Loop's subdivision for triangular meshes was combined with the diffusion model to arrive at a discretized version of the diffusion problem [1]. Mixed finite element methods based on surface subdivision technology were used to construct high-order smooth surfaces with specified boundary conditions in [14–16].

Subdivision surfaces can be compatible with NURBS as the standard in CAD systems which are capable of the refinability for B-spline techniques. The geometry models can be refined to arrive at a satisfactory accuracy of the numerical simulation where the subdivision schemes are simple, efficient and can be applied to meshes with arbitrary topology. However, it has not gained actual and extensive application in engineering. The principal difficulty is the exact and fast evaluation of the subdivision surfaces at arbitrary parameter values. Fortunately, there are some pioneering works about them [18,19].

There recently have been a few works on the application of subdivision methods in IGA. Volumetric IGA based on Catmull–Clark solids was investigated in [27]. For the IGA methods over complex physical domain, Powell–Sabin splines were used as IGA tools for advection–diffusion–reaction problems [29]. The bivariate splines in the rational Bernstein–Bézier form over the triangulation was applied in IGA [30]. A reproducing kernel triangular B-spline-based finite element method was proposed to solve PDEs [31].

Contributions. In this paper, we consider planar geometries with sharp/smooth boundaries and holes, and their control meshes have arbitrary topology. We adopt the finite element function space induced from the extended Loop's subdivision technique to represent both geometry and the solution space. Using this strategy, we are able to handle domains with arbitrary shaped boundary represented by piecewise cubic B-spline curves, which are the standard output of the modern CAD software systems. We establish the approximation properties of the limit formation of the extended Loop's subdivision. Using Poisson equation with the Dirichlet boundary condition as the numerical simulation model which corroborate the theoretical results, the IGA based on the extended Loop's subdivision can be naturally integrated into the framework of the standard FEA. Through the detailed numerical experiments, we demonstrate its efficiency, accuracy and robustness with the comparison to the classical FEA with piecewise linear elements.

The paper is organized as follows: Section 2 briefly reviews Loop's subdivision scheme including the extension around boundaries, and Stam's fast evaluation strategy for the limit surface of the subdivision. Section 3 describes IGA based on the extended Loop's subdivision using the Poisson equation with the Dirichlet boundary condition. In Section 4 we establish the approximation properties with the aid of the Bramble–Hilbert lemma. Section 5 shows detailed numerical experiments with the comparison to the FEA with linear elements. Section 6 is the conclusion and future work.

2. Extended Loop's subdivision for surfaces

The proposed IGA method is based on the extended Loop's subdivision technique for surfaces. In this section, we briefly review Loop's subdivision scheme, its extended scheme, and Stam's fast evaluation strategy for the limit surfaces of the subdivision.

2.1. Loop's subdivision scheme

Surface subdivision provides a simple, efficient algorithm to represent arbitrary topological free-form surfaces. A given surface mesh can be used to define a smooth surface by the limit of the subdivision process. Several subdivision schemes for generating smooth surfaces have been proposed. Some of them are interpolatory, i.e., the vertex positions of the coarse mesh are fixed, and only the newly added vertex positions need to be computed (see [8] for quadrilateral meshes, and [6,26] for triangular meshes), while others are approximatory (see [3,5] for quadrilateral meshes, [13] for triangular meshes, and [17] for general polyhedra meshes). These approximatory subdivision schemes compute both old and new vertex positions at each refinement step.

Loop proposed a subdivision scheme [13] for triangular meshes. The limit of this process is a smooth surface that is C^2-continuous except at a finite number of extraordinary vertices where the surface is C^1-continuous. This process subdivides each triangle into four sub-triangles. The original Loop's subdivision is only applied to closed meshes which includes the rules of vertex recomputation and edge insertion points. Consider a vertex $\mathbf{x}_0^k$ at level k with neighbor vertices $\mathbf{x}_i^k$ for $i = 1, \cdots, n$, where n is the valence of vertex $\mathbf{x}_0^k$. The old vertex is updated according to $\mathbf{x}_0^{k+1} = (1 - n\alpha)\mathbf{x}_0^k + \alpha\left(\mathbf{x}_1^k + \mathbf{x}_2^k + \cdots + \mathbf{x}_n^k\right)$ where $\alpha = \frac{1}{n}\left[\frac{5}{8} - \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2\right]$. Let $\mathbf{x}_l$ and $\mathbf{x}_r$ be the two wing neighbor vertices of edge $[\mathbf{x}_i\mathbf{x}_j]$, then the newly added vertex on this edge is defined as $\frac{3}{4}\mathbf{x}_i + \frac{3}{4}\mathbf{x}_j + \frac{1}{8}\mathbf{x}_l + \frac{1}{8}\mathbf{x}_r$.