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A novel three-dimensional mesh deformation method based on sphere relaxation

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ABSTRACT

In our previous work (2013) [19], we developed a disk relaxation based mesh deformation method for two-dimensional mesh deformation. In this paper, the idea of the disk relaxation is extended to the sphere relaxation for three-dimensional meshes with large deformations. We develop a node based pre-displacement procedure to apply initial movements on nodes according to their layer indices. Afterwards, the nodes are moved locally by the improved sphere relaxation algorithm to transfer boundary deformations and increase the mesh quality. A three-dimensional mesh smoothing method is also adopted to prevent the occurrence of the negative volume of elements, and further improve the mesh quality. Numerical applications in three-dimension including the wing rotation, bending beam and morphing aircraft are carried out. The results demonstrate that the sphere relaxation based approach generates the deformed mesh with high quality, especially regarding complex boundaries and large deformations.

1. Introduction

Dynamic mesh is crucial to many engineering computations, e.g., the numerical simulation of fluid–structure interaction, aeroelastic deformation and aerodynamic shape optimization, which usually involve moving or deforming boundaries. A popular method to generate dynamic meshes is the mesh deformation which adapts the computational mesh to the new deformed domain without changing the grid connectivity. Mesh deformation methods have received much attention in recent years, for they avoid mesh regeneration which is computational burdensome and introduces interpolation errors. The methods for mesh deformation can be generally sorted into partial differential equation (PDE) methods [1,2], physical analogy techniques [3,4], algebraic methods [6–12] and their combinations [5]. Algebraic methods are generally more efficient than the others, and become prevailing recently. The radial basis function (RBF) interpolation [6,7], the inverse distance weighting (IDW) [8,9] and the Delaunay interpolation [10] are the three common algebraic methods, which generate deformed mesh successfully for typical deformation patterns. Considerable new schemes and improvements of mesh deformation also have been developed recently. The octree decomposition was presented with the RBF interpolation to improve the deformation efficiency of large scale meshes [11]. Even an artificial neural network had been introduced with the interpolation technique to improve the efficiency of the mesh deformation method [12].

However, most mesh deformation approaches suffer from robustness problem when perform large deformations, especially those involving the relative motion of multi-boundaries. Furthermore, the initial mesh with low quality may fail in

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both small and large mesh deformations. The applications of mesh deformation in three-dimension (3D) are often troubled with these two difficulties, and arise challenges to the existed mesh deformation methods. The deformation of 3D meshes has been studied in the literature. Degand and Farhat [13] introduced the torsional springs on an additional triangle face to prevent the node from crossing its opposite facet in a tetrahedral element. Bottasso et al. [14] improved the spring analogy method by adding linear springs connecting each vertex with the center of its opposite facet in the tetrahedral element. Zhang et al. [15] combined the spring analogy with a wall distance function to generate dynamic meshes efficiently for 3D unsteady incompressible flows. The spring analogy was also combined with the partial differential equation method [16] to reduce the computational cost in 3D. Finite macro-element based method [17] was integrated with the transfinite interpolation to improve the efficiency of the mesh deformation in 3D. Nevertheless, these improved methods still have limitations in the applications of large deformations and complex boundaries, which may not maintain high mesh quality near the deformed boundaries.

Several mesh deformation approaches have been developed to preserve the high quality of deformed meshes near the moving boundaries in large deformations. Recently, Luke et al. [18] improved the deformed mesh quality as well as the efficiency of the IDW method using the tree code optimization. Zhou and Li [19] presented the disk relaxation algorithm (DRA) with a background mesh and mesh smoothing to produce the deformed mesh with high quality in large deformations. The DRA moves the nodes by three steps: first, the boundary deformation is transferred into the interior domain efficiently and smoothly by the background mesh; second, the mesh nodes are moved locally by disk relaxations to further spread the deformation and maintain high mesh quality; finally, a mesh smoothing technique is applied to keep the positive volume of the elements in the deformation procedure. The DRA had been proved to be quite successful for complex two-dimensional structures in large deformations [19]. The method has several significant advantages: first, the triangular elements of the deformed mesh have high mesh quality due to tangent disks; second, the mesh quality near the deformed boundaries is also preserved well by disk relaxations; third, the DRA even gives better deformed mesh quality than the initial computational mesh. Nevertheless, the DRA may not be applied directly to 3D applications since (a) it may be inapplicable to distribute the pre-displacement smoothly by the background mesh in 3D, especially for complex boundaries in irregular deformations; (b) the disk relaxation in DRA fails to guarantee the high mesh quality of tetrahedral elements in 3D; (c) the mesh smoothing technique in the DRA has limitations in 3D cases.

Sphere packing has been used in the mesh generation [20], for the compacted distribution of spheres preserves tetrahedral elements in high quality. Lo and Wang [21] packed the spheres tightly together by means of the advancing front approach to generate a set of ideal locations for the Delaunay point insertion. The sphere relaxation algorithm proposed by He et al. [22] has been applied in the random packing simulation of spheres [23]. The approach is modified to generate the deformed 3D mesh in this work.

The main objective of this work is to extend the idea of the DRA into 3D and inherit the good performance of the DRA in the applications of large deformations and complex boundaries. The proposed mesh deformation method is based on the sphere relaxation algorithm with pre-displacement and post-smoothing (SRA). First, the SRA moves nodes in the computational mesh roughly by the layer index of nodes. Second, a modified sphere relaxation algorithm is conducted to adjust the nodes locally. Finally, a mesh smoothing technique is applied to avoid the negative volume of the tetrahedral elements. 3D examples including the wing rotation, the bending beam and the morphing aircraft are presented in this work to demonstrate the deformation ability of the SRA.

2. Definitions and notations

In this section, we summarize and illustrate some important concepts and terms introduced in this work for clarity.

2.1. Sub-boundaries

A deformed boundary can be divided into several sub-boundaries according to their deformation patterns. For example, if one part of the boundary translates along the positive direction of an axis while the other moves along the opposite direction, then the boundary can be divided into two sub-boundaries. As shown in Fig. 1, the deformed boundary is considered separately as boundary I and II.

2.2. The layer index of an interior node and the reference boundary node

An interior node refers to the node placed in the computational domain except boundaries. The layer index of an interior node is the minimum number of mesh nodes connecting from a boundary node to the interior node. The specific boundary node is termed the reference boundary node of the interior node. Consider the mesh shown in Fig. 2, there are at least two nodes connecting from s_1 or s_2 to node p and three nodes from s_3 to node p. Therefore, the layer index of node p is 2, and the reference boundary node is chosen randomly between s_1 and s_2. In this example, we choose s_1 as the reference boundary node of the interior node p. s_1 can be considered as the nearest boundary node of node p with no searching needed.

For multi-bodies or sub-boundaries, an interior node has multiple layer indices referred to each boundary, as shown in Fig. 1. p is an interior node which has three layer indices corresponding to the deformed boundary I, II and the fixed outer