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Acoustic multiple scattering using recursive algorithms

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ABSTRACT

Acoustic multiple scattering by a cluster of cylinders in an acoustic medium is considered. A fast recursive technique is described which takes advantage of the multilevel Block Toeplitz structure of the linear system. A parallelization technique is described that enables efficient application of the proposed recursive algorithm for solving multilevel Block Toeplitz systems on high performance computer clusters. Numerical comparisons of CPU time and total elapsed time taken to solve the linear system using the direct LAPACK and TOEPLITZ libraries on Intel FORTRAN, show the advantage of the TOEPLITZ solver. Computations are optimized by multi-threading which displays improved efficiency of the TOEPLITZ solver with the increase of the number of scatterers and frequency.

1. Introduction

Multiple scattering (MS) and radiation of waves by a system of scatterers is of great theoretical and practical importance. MS is required in a wide variety of physical contexts such as the implementation of "invisibility" cloaks, the characterization of effective parameters of heterogeneous media, and the fabrication of dynamically tunable structures, i.e. superlenses and waveguides, etc. Our interest here is with examining acoustic MS from 2D cylindrical structures, although the method may be extended to 3D to include elastodynamic [1] or electromagnetic material properties. A broad review of the literature on single and MS and of the concepts of MS is given in [2]; a survey of more recent findings on MS from obstacles in acoustic and elastic media is provided in [1]. The development of numerically efficient techniques and algorithms that are appropriate for a wide range of problems is one of the main challenges in wave propagation research. The expensive costs of direct matrix inversion of a linear system motivates development of alternative numerically efficient methods [31]. Here we consider a recursive technique that is not limited by physical parameters such as the frequency or spacing between the scatterers, but is based on the structure of the MS formulation. The exact approach described in this paper takes advantage of multilevel Block Toeplitz structure of the linear system to speed up the matrix solution in a manner suitable for parallel computation. The recursive algorithm is robust, and resistant to machine errors.

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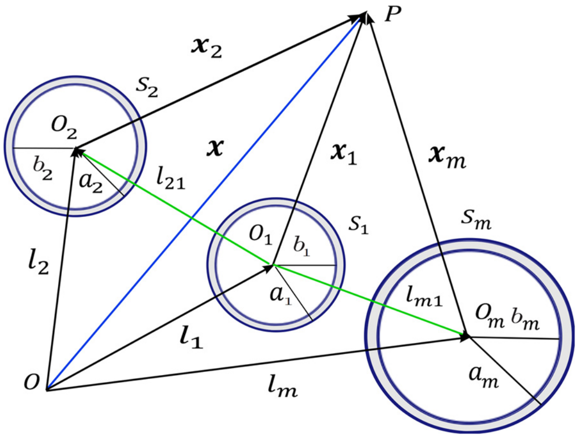

Fig. 1. An arbitrary planar configuration of M cylinders S_m with outer radius a_m and inner radius b_m, $m = \overline{1, M}$.

1.1. Problem definition

The two-dimensional (2D) MS problem may be reduced using Graf's addition theorem [3, eq. (9.1.79)] to an infinite linear system of equations which can be truncated to the finite dimensional system of the form:

$$\mathbb{X}\mathbf{b} = \mathbf{a}. \tag{1.1}$$

In this equation $\mathbf{a}$ is the column vector of the known coefficients of the excitation field, $\mathbf{b}$ is the column vector of the unknown scattering coefficients, and $\mathbb{X}$ is the interaction matrix that defines the coupling between each scatterer of the configuration (see Appendix A for details):

$$\mathbb{X} = \begin{bmatrix} \mathbf{I} & -\mathbf{T}^{(1)}\mathbf{P}^{1,2} & -\mathbf{T}^{(1)}\mathbf{P}^{1,3} & \cdots & -\mathbf{T}^{(1)}\mathbf{P}^{1,M} \\ -\mathbf{T}^{(2)}\mathbf{P}^{2,1} & \mathbf{I} & -\mathbf{T}^{(2)}\mathbf{P}^{2,3} & \cdots & -\mathbf{T}^{(2)}\mathbf{P}^{2,M} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -\mathbf{T}^{(M)}\mathbf{P}^{M,1} & -\mathbf{T}^{(M)}\mathbf{P}^{M,2} & -\mathbf{T}^{(M)}\mathbf{P}^{M,3} & \cdots & \mathbf{I} \end{bmatrix}. \tag{1.2}$$

The matrix $\mathbf{T}^{(j)}$ is the transition or T-matrix for scatterer j. The matrix $\mathbf{P}^{j,m} = [\mathbf{P}^{j,m}_{ql}]$ is a Toeplitz matrix; it depends on the position vector $\boldsymbol{l}_{jm}$ depicted in Fig. 1, and takes into account the interaction between the scatterers, whereas the transition matrix $\mathbf{T}^{(j)}$ depends on the shape and the physical properties of the material of cylinder, as well as the boundary conditions on the interfaces.

Here we consider 2-dimensional configurations of circularly cylindrical scatterers, for which the T-matrices become diagonal, see [1] for specific details. In particular, $\mathbf{P}^{j,m} = [\mathbf{P}^{j,m}_{ql}]$ $(q, l \in \mathbb{Z})$, where $\mathbf{P}^{j,m}_{ql} = V^{+}_{l-q}(\boldsymbol{l}_{jm})$ $(q = \overline{-N_j, N_j}, \ l = \overline{-N_m, N_m})$ $(j, m = \overline{1, M}, \ j \neq m)$, where the functions $V^{\pm}_n(\mathbf{x})$ are

$$V^{\pm}_n(\mathbf{x}) = H^{(1)}_n(k|\mathbf{x}|)e^{\pm in \arg \mathbf{x}}. \tag{1.3}$$

Here $H^{(1)}_n$ is the Hankel function of the first kind of order n, $\mathbf{x}$ is the position vector of point P (see Fig. 1), $k = \omega/c$ is the wavenumber, c is the acoustic speed, ω is the frequency with time dependence $e^{-i\omega t}$ assumed. The vectors in eq. (1.1) then have the structure

$$\mathbf{a} = \begin{pmatrix} \mathbf{T}^{(1)}\mathbf{a}^{(1)} \\ \mathbf{T}^{(2)}\mathbf{a}^{(2)} \\ \vdots \\ \mathbf{T}^{(M)}\mathbf{a}^{(M)} \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} \mathbf{b}^{(1)} \\ \mathbf{b}^{(2)} \\ \vdots \\ \mathbf{b}^{(M)} \end{pmatrix}, \quad \mathbf{a}^{(j)} = \begin{pmatrix} A^{(j)}_{-N} \\ A^{(j)}_{-N+1} \\ \vdots \\ A^{(j)}_{N} \end{pmatrix}, \quad \mathbf{b}^{(j)} = \begin{pmatrix} B^{(j)}_{-N} \\ B^{(j)}_{-N+1} \\ \vdots \\ B^{(j)}_{N} \end{pmatrix} \quad (j = \overline{1, M}), \tag{1.4}$$

where $\mathbf{a}^{(j)}$ $(j = \overline{1, M})$ is the vector of coefficients of the excitation field around cylinder S_j (see Fig. 1).

The matrix $\mathbb{X}$ is a complex valued dense $\mathtt{N} \times \mathtt{N}$ matrix, and $\mathtt{N}$ is proportional to the number of scatterers M multiplied by $2N + 1$ where N is the mode number. For high frequencies and a large number of scatterers, the system (1.1) becomes an extremely large linear system. The computational complexity of inverting $\mathbb{X}$ by direct methods is $O(\mathtt{N}^3)$, i.e. the Gauss–Jordan method requires $\mathtt{N}^3$ multiplication operations and $\mathtt{N}^3$ addition–subtraction operations, the Gauss method using *LU* decomposition requires $\mathtt{N}^3/3$ multiplication and $\mathtt{N}^3/3$ addition–subtraction operations. The memory requirements to solve (1.1) by direct methods grow as $O(\mathtt{N}^2)$. This is prohibitive for many realistic multiple scattering problems at high frequencies and a large number of scatterers. For large $\mathtt{N}$ and required number of iterations, the most widely used Krylov Space