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Divergence-free approximate Riemann solver for the quasi-neutral two-fluid plasma model

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ABSTRACT

A numerical method for the quasi-neutral two-fluid (QNTF) plasma model is described. The basic equations are ion and electron fluid equations and the Maxwell equations without displacement current. The neglect of displacement current is consistent with the assumption of charge neutrality. Therefore, Langmuir waves and electromagnetic waves are eliminated from the system, which is in clear contrast to the fully electromagnetic two-fluid model. It thus reduces to the ideal magnetohydrodynamic (MHD) equations in the long wavelength limit, but the two-fluid effect appearing at ion and electron inertial scales is fully taken into account. It is shown that the basic equations may be rewritten in a form that has formally the same structure as the MHD equations. The total mass, momentum, and energy are all written in the conservative form. A new three-dimensional numerical simulation code has been developed for the QNTF equations. The HLL (Harten–Lax–van Leer) approximate Riemann solver combined with the upwind constrained transport (UCT) scheme is applied. The method was originally developed for MHD [25], but works quite well for the present model as well. The simulation code is able to capture sharp multidimensional discontinuities as well as dispersive waves arising from the two-fluid effect at small scales without producing $\nabla \cdot \mathbf{B}$ errors. It is well known that conventional Hall-MHD codes often suffer a numerical stability issue associated with short wavelength whistler waves. On the other hand, since finite electron inertia introduces an upper bound to the phase speed of whistler waves in the present model, our code is free from the issue even without explicit dissipation terms or implicit time integration. Numerical experiments have confirmed that there is no need to resolve characteristic time scales such as plasma frequency or cyclotron frequency for numerical stability. Consequently, the QNTF model offers a better alternative to the Hall-MHD or fully electromagnetic two-fluid models.

1. Introduction

Understanding of a rich variety of nonlinear phenomena in space, astrophysical, and laboratory plasmas requires numerical simulations at various levels of approximations. At the largest scale, magnetohydrodynamic (MHD) description is useful because the scale-free nature of MHD allows us to conduct simulations with a realistic scale size. On the other hand, physics at kinetic scales (i.e., ion and electron inertial lengths) must be taken into account in cases where it plays the key role. A well-known example is the diffusion region of collisionless magnetic reconnection, in which the kinetic effect is

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essential for violating the frozen-in condition. Fully kinetic particle-in-cell (PIC) simulations have been used to investigate such problems. It is important to point out that the characteristics of spatially localized tiny regions may ultimately affect even the global dynamics of the system.

Although it is believed that physics beyond MHD ultimately needs to be incorporated properly even for the modeling of macroscopic phenomena, it is still a formidable task, albeit not impossible, to perform fully kinetic PIC simulations at a macroscopic scale. In practice, it is desirable to start with a simpler model and gradually proceed toward better (but more complicated) physics models with less approximations. Hall-MHD is one of such better models in the sense that it takes into account physics at the ion inertial scale. The hybrid simulation model that deals with kinetic ions and a massless fluid electron can be considered as a "kinetic version" of Hall-MHD. The Hall-MHD and hybrid models are therefore believed to be possible alternatives to MHD for the next generation global modeling.

Although both Hall-MHD and hybrid have been well established standard models for simulations of collisionless plasmas, it is well known that they often suffer a numerical difficulty due to high frequency whistler waves. The dispersion relation of whistler waves $\omega \propto k^2$ leads to the increase in the phase speed at short wavelength without bound. This is generally thought of as a source of numerical instability. A common strategy to stabilize such simulations is to introduce ad-hoc numerical dissipation such as hyper-resistivity in the code [27,35]. However, it is not easy to control the amount of numerical dissipation with this kind of approach. Furthermore, since the strategy is quite different from the philosophy of modern high-order shock capturing schemes, this makes it difficult to extend such codes to the Hall-MHD regime. Although one may use an implicit scheme to circumvent the problem, this will make implementation of the algorithm much more complex (e.g., [3,42]).

A more straightforward approach is to employ the fully electromagnetic two-fluid (EMTF) plasma model in which the full set of Maxwell equations are coupled with two separate (i.e., ion and electron) fluids equations [37,26,19,38,20]. The phase speed of whistler waves has an upper bound in this system due to the presence of finite electron inertia. On the other hand, since it is essentially a fluid counterpart of the PIC simulation, it must deal with high frequency Langmuir waves as well as electromagnetic waves. Numerical stability requires that these waves should adequately be resolved by the simulation time step unless more complicated implicit schemes are employed [20]. In general, however, these high frequency waves are not of interest as far as macroscopic dynamics is concerned. The neglect of displacement current (which implies the charge neutrality) in the Maxwell equations is indeed a reasonable assumption if one considers non-relativistic problems, although this does not in general apply to highly relativistic plasmas (e.g., [2]).

There is also a way to incorporate finite electron inertia effect into Hall-MHD/hybrid without resorting to the full set of Maxwell equations. Conventionally, finite electron inertia effect has been included as a correction to the magnetic field (e.g., [23,34,35,31]). Although most of previous studies adopted some kind of simplification, the finite electron inertia effect if appropriately included can correctly introduce an upper bound to the phase speed of whistler waves. On the other hand, the motivation for these studies was to initiate spontaneous magnetic reconnection without relying on an anomalous resistivity model. Therefore, a possible advantage of finite electron inertia effect on the numerical stability issue has not been paid much attention. We have recently shown that, by modifying the procedure to incorporate finite electron inertia into the model, hybrid simulations can be made much more robust particularly in low-density regions where whistlers become problematic [1]. This was made possible by implementing finite electron inertia as a correction to the electric field (i.e., the generalized Ohm's law), which is then used to update the magnetic field. This physically more consistent approach gives a natural way to handle even a pure vacuum region in a hybrid code. It is quite natural to expect that essentially the same methodology can be applied to Hall-MHD equations, because kinetic ion dynamics does not play a role for dispersion of whistler waves.

In the present paper, we consider a system consisting of two-fluid equations coupled with Maxwell equation without displacement current (Darwin approximation), which we call the quasi-neutral two-fluid (QNTF) model. As we will see in the next section, it is approximately the same as the Hall-MHD equations with finite electron inertia, but terms dropped in previous studies are retained for consistency. Consequently, the total mass, momentum, and energy including finite electron contributions are all written in the conservative form. The conservation laws coupled with the induction equation for the magnetic field have the same formal structure as the MHD equations, which thus may be solved by a known conservative scheme. Because of the neglect of displacement current, there are no high frequency waves such as Langmuir or electromagnetic waves, and the number of eigenmodes is indeed the same as MHD. The system correctly reduces to the ideal MHD in the long wavelength limit. Therefore, we think that it provides a natural extension of MHD having desirable properties both in terms of physics and numerics.

We have developed a three-dimensional (3D) numerical simulation code to solve the proposed system of equations. We employ the single-state HLL (Harten–Lax–van Leer) approximate Riemann solver as a building block. It only requires the maximum characteristic speed, and is independent of detailed information on the eigenmode structure. The scheme is thus suitable to the QNTF equations because eigenmode decomposition for this system should certainly be much more laborious task than for MHD. In addition, we adopt the Upwind Constrained Transport (UCT) scheme to keep the divergence error of the magnetic field within machine accuracy [25]. The UCT scheme is based on the Constrained Transport (CT) scheme [16], but is designed specifically to be consistent with an underlying Riemann solver. Although it was originally developed for MHD, we found it is useful for the QNTF equations as well. With these numerical techniques, our simulation code is able to capture sharp discontinuities as well as dispersive waves arising from the two-fluid effect at the same time even in multidimensions without violating the divergence-free property.