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Lattice Boltzmann model for collisionless electrostatic drift wave turbulence obeying Charney–Hasegawa–Mima dynamics

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ABSTRACT

A lattice Boltzmann method (LBM) approach to the Charney–Hasegawa–Mima (CHM) model for adiabatic drift wave turbulence in magnetised plasmas is implemented. The CHM-LBM model contains a barotropic equation of state for the potential, a force term including a cross-product analogous to the Coriolis force in quasigeostrophic models, and a density gradient source term. Expansion of the resulting lattice Boltzmann model equations leads to cold-ion fluid continuity and momentum equations, which resemble CHM dynamics under drift ordering. The resulting numerical solutions of standard test cases (monopole propagation, stable drift modes and decaying turbulence) are compared to results obtained by a conventional finite difference scheme that directly discretizes the CHM equation. The LB scheme resembles characteristic CHM dynamics apart from an additional shear in the density gradient direction. The occurring shear reduces with the drift ratio and is ascribed to the compressible limit of the underlying LBM.

1. Introduction

The lattice Boltzmann method (LBM) has been established as a promising tool for computations in fluid dynamics, including turbulence, reactive and complex flows. The LB method to model fluid partial differential equations in the framework of a reduced discrete kinetic theory has also been applied to plasma physics. Problems like magnetohydrodynamic turbulence (treated for example in Refs. [1–14]), magnetic reconnection [15–17], and a first approach to electrostatic turbulence [18] have been addressed in this framework.

The Charney–Hasegawa–Mima (CHM) equation serves as a basic prototypical two-dimensional one-field model for collisionless electrostatic drift wave turbulence in magnetised plasmas with cold ions and isothermal electrons with an adiabatic response. Drift wave turbulence taps free energy from the background plasma pressure gradient to drive advective nonlinear motion of pressure disturbances by the $E \times B$ drift velocity perpendicular to the magnetic field $\boldsymbol{B}$. Parallel dynamics are captured by the electron currents which are balancing the pressure deviations electrostatically with an adiabatic response

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along the magnetic field. The spatial scale is highly anisotropic permitting us to decouple the parallel dynamics from the perpendicular drift plane motion obeying the two-dimensional (normalised) CHM equation [19,20]

$$\left(1-\nabla^2\right)\frac{\partial\delta\phi}{\partial t}+\frac{\partial\delta\phi}{\partial y}-\left\{\delta\phi,\nabla^2\delta\phi\right\}=0 \tag{1}$$

where the advective nonlinearity is expressed by a Poisson bracket $\{A,B\}=\partial_x A\,\partial_y B-\partial_y A\,\partial_x B$. The equation is normalised according to $\boldsymbol{x}\leftarrow\boldsymbol{x}/\rho_s$ and $t\leftarrow\kappa_n\omega_{ci}t$ for the length and time scales and fluctuations $\delta\phi\leftarrow\kappa_n^{-1}(e\phi/T_e)$ for the electrostatic potential ϕ. These scales represent the dominant contributions to turbulent transport in magnetised plasmas, where the drift frequency $\omega\sim(\rho_s/L_n)\omega_{ci}$ appears to be lower in magnitude then the ion gyro-frequency $\omega_{ci}=c_s/\rho_s$ describing the gyro-motion of ions around the magnetic field lines. The magnitude is specified by the ratio of the drift scale $\rho_s=\sqrt{m_iT_e}/eB$ (corresponding to a gyro-radius of ions of mass m_i at electron temperature T_e) to the gradient length $L_n=|\partial_x\ln n_0(x)|^{-1}$ of the static background density $n_0(x)$ and is typically defined by the drift ratio $\kappa_n=\rho_s/L_n\ll 1$. The sound speed $c_s=\sqrt{T_e/m_i}$ is given in terms of the electron temperature and ion mass. Finite ion temperature ($T_i>0$) effects arise when the ion gyro-radius $\rho_i=\sqrt{m_iT_i}/eB$ approaches typical fluctuation scales and are beyond the scope of the model. More detailed gyrokinetic or gyrofluid models put emphasise on accurate averaging procedures over gyro-motion and modifications to the polarization equation [21–23].

The CHM equation can be either obtained from a gyrokinetic model, or from the continuity and momentum equations for a cold uniformally magnetised ion fluid ($T_i\ll T_e$) with adiabatic electron response and a negative background density gradient in x-direction, $n_i=n_e=n_0(x)\exp[e\phi/kT_e]$. The normalised ion continuity and momentum equations can be expressed in terms of the potential instead of density [24] as

$$\kappa_n\frac{d}{dt}\delta\phi+\nabla\cdot\boldsymbol{u}=\kappa_n\boldsymbol{u}\cdot\nabla x \tag{2}$$

$$\kappa_n\frac{d}{dt}\boldsymbol{u}+\boldsymbol{e}_z\times\boldsymbol{u}=-\nabla\delta\phi \tag{3}$$

where $d/dt=\partial_t+\boldsymbol{u}\cdot\nabla$ is the advective derivative. Expanding $\delta\phi$ and $\boldsymbol{u}$ in an asymptotic series with the drift ratio $\kappa_n\ll 1$ as small expansion parameter and accordant ordering [24] yields the CHM equation (1). Replacing the drift ratio κ_n with the Rossby number Ro and identifying the electrostatic potential fluctuations with the dimensionless surface height reveals the isomorphism to the quasi-geostrophic single layer shallow water equations in the β-plane approximation. By replacing the density gradient with a bottom topography or a spatially varying Coriolis frequency the CHM equation is resembled in the limit of a small Rossby number $Ro\ll 1$. Advances with the lattice Boltzmann method to the shallow water equations have been made by Zhong et al. [25–27] and Dellar [28].

2. Lattice Boltzmann model

2.1. Boltzmann equation

Starting point for the lattice discretisation is the Boltzmann equation for the kinetic distribution function $f(\boldsymbol{x},\boldsymbol{\xi},t)$ with a Bhatnagar–Gross–Krook (BGK) collision operator $C=-(f-f^{eq})/\tau_c$, which expresses the relaxation to a local Maxwellian for a time constant τ_c. Applying the diffusive scaling $t\to t/\epsilon^2$ and $\boldsymbol{x}\to\boldsymbol{x}/\epsilon$ on the Boltzmann equation results in its dimensionless form [29]

$$\frac{\partial}{\partial t}f+\frac{1}{\epsilon}\boldsymbol{\xi}\cdot\nabla f=\frac{1}{\epsilon^2}\left[A(f-f^{eq})+F\right], \tag{4}$$

where source and force terms are included in a forcing function as $F(\boldsymbol{x},\boldsymbol{\xi},t)=-\boldsymbol{a}\cdot\nabla_{\boldsymbol{\xi}}f(\boldsymbol{x},\boldsymbol{\xi},t)+s(\boldsymbol{x},\boldsymbol{\xi},t)$ and the single time collision operator is defined by $A=-1/(\epsilon\tau)$.

The Knudsen number $\epsilon=\lambda_m/L_0$ and the non-dimensional relaxation time $\tau=\tau_c/t_c$ are here defined in relation to characteristic drift scale $L_0=\rho_s$ and to the collision time $t_c=\lambda_m/U_0$ with mean free path length $\lambda_m=e_m\tau_c$ and characteristic (drift) velocity $U_0=\kappa_nc_s$. The dimensionless relaxation time $\tau=U_0/e_m$ relates the flow velocity to the (lattice) molecular velocity e_m whereas the Mach number $Ma=U_0/c_s$ is identified with the drift parameter κ_n.

The dynamics in the fluid limit, given by eqs. (2) and (3), can be consistently described with the kinetic equation (4) assuming a local Maxwellian equilibrium distribution function of the form [6]

$$f^{eq}=\frac{\phi}{(2\pi\Theta)^{D/2}}\exp\left[-\frac{(\boldsymbol{\xi}-\boldsymbol{u})^2}{2\Theta}\right]. \tag{5}$$

The squared dimensionless barotropic speed of sound $\Theta=\phi/(2\kappa_n^2)$ results from the barotropic pressure term $P=\phi^2/(2\kappa_n^2)$ appearing on the macroscopic level as in eq. (3). The isothermal squared speed of sound is defined by $\theta=1/\kappa_n^2$.

Macroscopic quantities are defined by taking velocity moments over the distribution function