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Nonlinear analysis of multiphase transport in porous media in the presence of viscous, buoyancy, and capillary forces

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ABSTRACT

Nonlinear convergence problems in numerical reservoir simulation can lead to unacceptably large computational time and are often the main impediment to performing simulation studies of large-scale problems. We analyze the nonlinearity of the discrete transport (mass conservation) equation for immiscible, incompressible, two-phase flow in porous media in the presence of viscous, buoyancy, and capillary forces. Although simulation problems are multi-dimensional with large numbers of cells and variables, we find that the essence of the nonlinear behavior can be understood by studying the discretized (numerical) flux function for the interface between two cells. The numerical flux is expressed in terms of the saturations of the two cells. Discontinuities in the first-order derivative of the flux function (referred to as kinks) and inflection lines are identified as the cause of convergence difficulty. These critical features (kinks and inflections) change the curvature of the numerical flux function abruptly, and can lead to overshoots, oscillations, or divergence in Newton iterations.

Based on our understanding of the nonlinearity, a nonlinear solver is developed, referred to as the Numerical Trust Region (NTR) solver. The solver is able to guide the Newton iterations safely and efficiently through the different saturation 'trust-regions' delineated by the kinks and inflections. Specifically, overshoots and oscillations that often lead to convergence failure are avoided. Numerical examples demonstrate that our NTR solver has superior convergence performance compared with existing methods. In particular, convergence is achieved for a wide range of timestep sizes and Courant–Friedrichs–Lewy (CFL) numbers spanning several orders of magnitude. In addition, a discretization scheme is proposed for handling heterogeneities in capillary-pressure–saturation relationship. The scheme has less degree of nonlinearity compared with the standard Single-point Phase-based Upstream weighting scheme, leading to an improved nonlinear convergence performance especially when used together with our NTR solver. Our proposed numerical solution strategy that is based on the numerical flux and handles capillarity extends the previous work by Jenny et al. (2009) [6] and Wang and Tchelepi (2013) [7] significantly.

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Nomenclature

Abbreviations

CFL	Courant–Friedrichs–Lewy number
CPVI	cell pore volumes injected
DIC	Decoupled Integral Capillarity
FIM	Fully Implicit Method
NTR	Numerical Trust Region
SIM	Sequential Implicit Method
SPU	Single-point Phase-based Upstream weighting

Constants

g	gravitational acceleration

Greek

α	Newton-update angle
β	Brooks–Corey coefficient
ε	size of the buffer zone
λ	phase mobility
λ_T	total mobility
μ	phase viscosity
ϕ	porosity
ρ	phase density
τ	characteristic time for gravity segregation
Θ	contraction factor
ξ	chopping ratio (also called damping factor)

Symbols

∇	divergence or gradient

Variables

C	integral form of capillarity
F	numerical flux
f	analytical flux
h	height
J	Jacobian matrix
k	permeability
k_r	relative permeability
L	length
N	number of cells in a simulation domain
N_g	gravity number
p	pressure
P_c	capillary pressure
$P_{c,e}$	capillary entry pressure
P_e	Peclet number
Q	mass flow rate of a phase
q	volumetric flow rate of a phase
R	residual
S	phase saturation
t	time
u	phase velocity
u_T	total velocity

Superscripts

k	iteration index
n	timestep index
$-$	left side of a cell interface
$+$	right side of a cell interface
$*$	dimensionless quantity

Subscripts

α	a particular phase, either wetting or nonwetting
c	capillary flux
D	downstream with respect to total velocity
inflect	inflection point
n	nonwetting phase
U	upstream with respect to total velocity
vg	viscous–buoyancy flux
w	wetting phase

1. Introduction

Numerical simulation is widely used to understand, predict, and manage subsurface fluid migration with applications to oil/gas recovery, groundwater remediation, and CO_2 geological sequestration. The reservoir models often have complex geometry with highly detailed descriptions of the heterogeneity, and the coupled conservation laws that describe the multiphase fluid flow and transport are highly nonlinear. Several methods are available to solve the conservation equations numerically [1]. The use of explicit time integration schemes generally poses severe restrictions on the timestep size, especially for large and heterogeneous simulation problems where the Courant–Friedrichs–Lewy (CFL) numbers can vary by several orders of magnitude in the domain [2]. Implicit time integration is usually preferred in these situations.

Implicit schemes such as the Fully Implicit Method (FIM) [1], or the Sequential Implicit Method (SIM) [1,3], usually solve the conservation equations (cast in residual form) using the Newton method. For a target timestep, a sequence of Newton iterations is performed until the solution of the nonlinear algebraic equations is obtained. Each iteration requires construction of the Jacobian matrix and solution of the resulting linear system. Due to the nonlinearity of the coupled conservation equations, the Newton method is not guaranteed to converge if the timestep is large [1]. When convergence fails within a specified computational effort, the Newton scheme is restarted with a smaller timestep, and the previous effort is wasted. The new smaller timestep is chosen heuristically. This heuristic timestep control technique is cumbersome and problem specific [1,4,5]. Even when such heuristics work, they tend to be conservative and can result in unnecessarily long simulation times.

Our objective is to develop an unconditionally convergent nonlinear solver for multiphase transport in porous media. Having such a capability not only will improve the computational speed of numerical simulations, but will also allow the timestep size to be selected based on accuracy considerations (time truncation error) as opposed to the ability of the