Journal of Computational Physics 297 (2015) 156–181

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Fully decoupled time-marching schemes for incompressible fluid/thin-walled structure interaction

Miguel A. Fernández [a,b,*], Mikel Landajuela [a,b], Marina Vidrascu [a,b]

[a] Inria Paris-Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France
[b] Sorbonne Universités, UPMC Univ. Paris 6, Lab. Jacques-Louis Lions, 4 place Jussieu, 75252 Paris Cedex 05, France

ARTICLE INFO

Article history:
Received 10 December 2014
Received in revised form 13 March 2015
Accepted 2 May 2015
Available online 13 May 2015

Keywords:
Fluid–structure interaction
Incompressible fluid
Reissner–Mindlin shell
Time-discretization
Projection method
Explicit coupling scheme

ABSTRACT

In this paper we introduce a class of fully decoupled time-marching schemes (velocity–pressure–displacement splitting) for the coupling of an incompressible fluid with a thin-walled elastic or viscoelastic structure. The time splitting combines a projection method in the fluid with a specific Robin–Neumann treatment of the interface coupling. A priori energy estimates guaranteeing unconditional stability are established for some of the schemes. The accuracy and performance of the methods proposed are illustrated by a thorough numerical study.

1. Introduction

Mathematical problems describing the mechanical interaction of an elastic thin-walled structure with an incompressible fluid flow appear in a wide variety of engineering fields: from the aeroelasticity of sailing boats and parachutes, to sloshing dynamics in tanks, heat exchangers design, micro-encapsulation technology and the biomechanics of animal cells and physiological flows (see, e.g., [1–8]).

Typically, these coupled problems (and other multi-physics problems in general) are discretized by exploiting the heterogeneous nature of the mechanical system. For instance, the fluid and the solid equations are often discretized by different time-stepping schemes adapted to their distinct mathematical properties. The time-discretization of the coupling conditions determines the so-called coupling scheme: implicit, semi-implicit and explicit (see, e.g., [9–11] for recent reviews).

With an implicit coupling scheme no time lag exits between the fluid and solid time-marchings. This can deliver unconditional stability and optimal accuracy, but at the price of solving a computationally demanding coupled problem at each time-step. The corresponding solution procedures are traditionally referred to in the literature as: monolithic and partitioned. Monolithic methods solve the coupled problem as a single system of equations (see, e.g., [12–16]). Partitioned methods, on the contrary, exploit the heterogeneous nature of the system via (recurrent) separate solutions of the fluid and solid equations, with appropriate interface conditions (see, e.g., [17–21]). Partitioned solution procedures are very appealing because of their intrinsic modularity, which enables the reuse of independent efficient solvers. Such an advantage comes however at a price, computational efficiency over a monolithic approach is not necessarily guaranteed (see, e.g., [12,14]).

* Corresponding author at: Inria Paris-Rocquencourt, BP 105, 78153 Le Chesnay Cedex, France.
E-mail address: miguel.fernandez@inria.fr (M.A. Fernández).

http://dx.doi.org/10.1016/j.jcp.2015.05.009
0021-9991/

Stable and less computationally onerous alternatives to implicit coupling are the so-called semi-implicit coupling schemes. These methods enforce a specific explicit/implicit treatment of the interface coupling conditions (see, e.g., [22,23]) and are often combined with a fractional-step time-marching in the fluid (see, e.g., [24–28]) or in the solid (see, e.g., [29–32]). The implicit part of the coupling (which, as above, can be solved in a monolithic or a partitioned fashion) guarantees stability, while the explicit one reduces computational complexity.

Explicit coupling schemes (also termed loosely coupled) uncouple the fluid and solid time-marchings via appropriate explicit discretizations of the interface conditions. The resulting solution procedures are thus naturally partitioned. The design and the analysis of stable and accurate explicit coupling schemes for incompressible fluid–structure interaction problem is a challenging problem. This is due to the fact that the interface coupling can be extremely stiff (see, e.g., [33]). Though stability has been an open problem for years (see [34]), the most intricate issue appears to be accuracy. The explicit coupling schemes reported in [34,35] guarantee stability but at the expense of a degradation of accuracy, which requires suitable correction iterations. In the case of the coupling with an elastic thin-walled solid, unconditional stability is achieved with the explicit coupling scheme introduced in [29], which is known to yield very poor accuracy (see [30,36]). Numerical evidence suggests that enhanced accuracy can be obtained with the variants recently reported in [31,32]. Unfortunately, if physical damping is present in the structure equations, these coupling schemes are no longer explicit. These issues are overcome by the Robin–Neumann based explicit coupling schemes proposed in [37], which simultaneously deliver unconditional stability and optimal (first-order) time accuracy. A fundamental ingredient in the stability and accuracy of these methods is a specific combination of the interface Robin consistency of the coupled problem with a monolithic time-stepping in the fluid.

Since the pioneering work by Chorin and Temam (see [38,39]), projection methods have become one of the most widespread techniques for the numerical solution of the incompressible Navier–Stokes equations in primitive variables (see, e.g., [40–43] and the references therein). These methods segregate the computation of the velocity and of the pressure in terms of two decoupled elliptic problems which make them very appealing for large scale computations. The contribution of the present paper is to show how the explicit Robin–Neumann paradigm of [37] can be effectively used with a projection based time-marching in the fluid. An approach in this direction, intended to deliver second-order accuracy, has been recently reported in [44]. It is however not clear how to implement the interface splitting therein within a finite element framework.

The key idea of the schemes proposed in this paper lies in the derivation of an intrinsic fractional-step time-stepping of the interface Robin consistency. This preserves the stability and accuracy of the original Robin–Neumann splitting without compromising the velocity/pressure uncoupling in the fluid time-marching. In particular, the resulting solution procedures enable a fully decoupled computation of the whole fluid–solid state. The velocity/pressure splitting in the fluid introduces additional perturbations of the kinematic coupling which make the analysis much more intricate than in [37]. For a linear coupled problem involving the Stokes equations and a general (*Reissner–Mindlin* type) viscoelastic shell model, a priori energy estimates guaranteeing unconditional stability are derived for some of the variants. The proposed fully decoupled schemes are also formulated within a non-linear framework, involving the incompressible Navier–Stokes equations (in moving domains) and a non-linear viscoelastic shell model. A thorough numerical study, based on different linear and non-linear fluid–structure interaction examples, illustrates the accuracy and performance of the methods proposed.

The rest of the paper is organized as follows. Section 2 is devoted to the derivation and the analysis of the methods within a linear representative setting. In Section 3, the proposed fully decoupled schemes are formulated within a non-linear setting. The numerical results are presented and discussed in Section 4. Finally, a summary of the conclusions and some directions of further investigation are given in Section 5.

Some preliminary results of the present work have been announced, without proof, in [45].

2. Derivation and analysis in the linear case

We consider a simplified linear model problem in which the fluid is described by the Stokes equations, in a fixed domain, and the structure by a linear viscoelastic *Reissner–Mindlin* shell model (see, e.g., [46,47]). Basically, we assume that the displacements of the shell are infinitesimal and that the Reynolds number in the fluid is small. We denote by $\Omega \subset \mathbb{R}^3$ the fluid domain and by $\partial\Omega$ its boundary. The fluid boundary is partitioned as $\partial\Omega = \Gamma \cup \Sigma$, where Σ stands for the fluid–structure interface. Since the structure is thin-walled, the interface Σ is itself the reference configuration of the shell mid-surface (see Remark 2.2 below). The exterior unit-vector normal to $\partial\Omega$ is denoted by $\boldsymbol{n}$. For a given vector field $\boldsymbol{v}$ defined on the surface Σ, the symbols $\boldsymbol{v}_\perp \stackrel{\text{def}}{=} (\boldsymbol{v}\cdot\boldsymbol{n})\boldsymbol{n}$ and $\boldsymbol{v}_\parallel \stackrel{\text{def}}{=} \boldsymbol{v} - \boldsymbol{v}_\perp$ will denote, respectively, the normal and tangential components of $\boldsymbol{v}$.

The resulting coupled problem reads as follows: find the fluid velocity $\boldsymbol{u} : \Omega \times \mathbb{R}^+ \to \mathbb{R}^3$, the pressure $p : \Omega \times \mathbb{R}^+ \to \mathbb{R}$, the solid displacement $\boldsymbol{d} : \Sigma \times \mathbb{R}^+ \to \mathbb{R}^3$ and the rotation vector $\boldsymbol{\theta} : \Sigma \times \mathbb{R}^+ \to \mathbb{R}^3$, satisfying the *Reissner–Mindlin* kinematical assumption $\boldsymbol{\theta}_\perp = \boldsymbol{0}$, such that

$$\begin{cases} \rho^{\mathrm{f}}\partial_t \boldsymbol{u} - \mathbf{div}\boldsymbol{\sigma}(\boldsymbol{u}, p) = \boldsymbol{0} & \text{in } \Omega, \\ \mathrm{div}\boldsymbol{u} = 0 & \text{in } \Omega, \\ \boldsymbol{\sigma}(\boldsymbol{u}, p)\boldsymbol{n} = -p_\Gamma \boldsymbol{n} & \text{on } \Gamma, \end{cases} \tag{1}$$