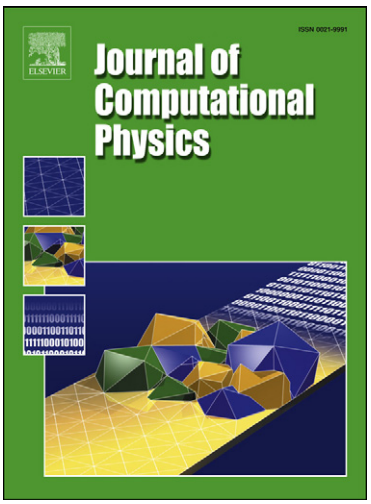

A finite element method to compute three-dimensional equilibrium configurations of fluid membranes: Optimal parameterization, variational formulation and applications

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A finite element method to compute three-dimensional equilibrium configurations of fluid membranes: optimal parameterization, variational formulation and applications

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Abstract

We introduce a finite element method to compute equilibrium configurations of fluid membranes, identified as stationary points of a curvature-dependent bending energy functional under certain geometric constraints. The reparameterization symmetries in the problem pose a challenge in designing parametric finite element methods, and existing methods commonly resort to Lagrange multipliers or penalty parameters. In contrast, we exploit these symmetries by representing solution surfaces as normal offsets of given reference surfaces and entirely bypass the need for artificial constraints. We then resort to a Galerkin finite element method to compute discrete C^1 approximations of the normal offset coordinate.

The variational framework presented is suitable for computing deformations of three-dimensional membranes subject to a broad range of external interactions. We provide a systematic algorithm for computing large deformations, wherein solutions at subsequent load steps are identified as perturbations of previously computed ones. We discuss the numerical implementation of the method in detail and demonstrate its optimal convergence properties using examples. We discuss applications of the method to studying adhesive interactions of fluid membranes with rigid substrates and to investigate the influence of membrane tension in tether formation.

Keywords: Biomembrane; cell adhesion; finite element method; cell-nanoparticle interaction; drug delivery

1. Introduction

1.1. Background

Lipid bilayers are fluid-like membranes and a principal constituent in the boundaries of virtually all cells and intracellular organelles. While real biological structures are quite complex, lipid bilayers serve as a simple yet representative medium to understand important mechanical and chemical functions of cell and organelle boundaries. We are concerned here with designing finite element methods to simulate the mechanical behavior of lipid bilayers modeled as fluid membranes.

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