



Iterative stability analysis of spatial domain decomposition based on block Jacobi algorithm for the diamond-difference scheme



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ABSTRACT

We study convergence of the integral transport matrix method (ITMM) based on a parallel block Jacobi (PBJ) iterative strategy for solving particle transport problems. The ITMM is a spatial domain decomposition method proposed for massively parallel computations. A Fourier analysis of the PBJ-based iterations applied to S_N diamond-difference equations in 1D slab and 2D Cartesian geometries is performed. It is carried out for infinite-medium problems with homogeneous material properties. To analyze the performance of the ITMM with the PBJ algorithm and evaluate its potential in scalability we consider a limiting case of one spatial cell per subdomain. The analysis shows that in such limit the spectral radius of the iteration method is one without regard to values of the scattering ratio and optical thickness of the spatial cells. This implies lack of convergence in infinite medium. Numerical results of finite-medium problems are presented. They demonstrate effects of finite size of spatial domain on the performance of the iteration algorithm as well as its asymptotic behavior when the extent of the spatial domain increases. These numerical experiments also show that for finite domains iterative convergence to a finite criterion is achievable in a multiple of the sum of number of cells in each dimension.

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1. Introduction

A wide variety of physical problems involves transport of particles in matter when they interact with a background medium but collisions between particles themselves has a negligible effect on the transport process. The mathematical description of such kind of particle transport phenomena is based on the linear Boltzmann equation. It is referred to as particle transport or radiative transfer equation depending on the underlying physical processes. These problems have a lot of applications in science and engineering [1,2]. The transport equation is a foundation for mathematical models of radiative transfer in plasmas and atmospheres, neutron transport in nuclear reactors and well-logging problems, photon and electron transport in cancer therapy problems.

The transport equation is a formulation of detailed particle balance in phase space. If particles are treated classically, then their distribution depends on spatial position (\mathbf{r}), direction of motion ($\boldsymbol{\Omega}$), energy (E), and time (t). The solution of the transport equation of various forms is related to the particle distribution function $n(\mathbf{r}, \boldsymbol{\Omega}, E, t)$. For example, the neutron

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transport equation is formulated for the particle angular flux $\psi = vn$, where v is the particle speed. The radiative transfer equation has a form similar to one for neutrons and defined for the specific intensity $I = ch\nu n$, where ν is the photon frequency and c is the speed of light. When one simulates processes in a physical system, the transport equation is just a part of a large system of nonlinearly coupled differential equations describing multiphysical phenomena. It enables one to model processes associated with dynamic energy redistribution in the system governed by particle transport and interaction with matter.

The steady-state one-group particle transport equation has the following form:

$$\mathcal{L}\psi(\mathbf{r}, \boldsymbol{\Omega}) = \mathcal{S}\psi(\mathbf{r}, \boldsymbol{\Omega}) + \frac{1}{4\pi}q(\mathbf{r}), \quad \mathbf{r} \in \mathbf{D}, \quad \text{for all } \boldsymbol{\Omega}, \tag{1}$$

$$\psi|_{\mathbf{r} \in \partial\mathbf{D}} = \psi^{in}(\mathbf{r}_\gamma, \boldsymbol{\Omega}), \quad \text{for } \mathbf{r}_\gamma \in \partial\mathbf{D} \text{ and } \boldsymbol{\Omega} \cdot \mathbf{e}_n < 0, \tag{2}$$

where \mathbf{D} is the spatial domain, \mathbf{e}_n is outward normal at the external boundary $\partial\mathbf{D}$. The operator \mathcal{L} accounts for particle streaming and collisions

$$\mathcal{L}\psi \equiv \boldsymbol{\Omega} \cdot \nabla\psi(\mathbf{r}, \boldsymbol{\Omega}) + \sigma_t(\mathbf{r})\psi(\mathbf{r}, \boldsymbol{\Omega}), \tag{3}$$

where σ_t is the total cross sections. The integral operator \mathcal{S} represents scattering processes and in case of isotropic scattering is given by

$$\mathcal{S}\psi \equiv \frac{1}{4\pi}\sigma_s(\mathbf{r}) \int_{4\pi} \psi(\mathbf{r}, \boldsymbol{\Omega})d\boldsymbol{\Omega} = \frac{1}{4\pi}\sigma_s(\mathbf{r})\phi(\mathbf{r}), \tag{4}$$

where σ_s is the scattering cross section and

$$\phi(\mathbf{r}) = \int_{4\pi} \psi(\mathbf{r}, \boldsymbol{\Omega})d\boldsymbol{\Omega} \tag{5}$$

is scalar flux. q is an isotropic source of particles. Except for highly idealized models, the transport equation can be solved only by means of numerical methods. The essential feature of the transport equation is that the phase space is of high dimensionality. In many practical applications discretization of the transport equation leads to a very large number of unknowns in the phase space. Computational transport problems with more than 10^{10} unknowns are rather common nowadays. To solve this type of numerical problems one relies on massively parallel computations using high-performance computers. Parallel methodologies for transport problems are based on various transport sweeping algorithms and domain decomposition methods [3–10].

Let us consider that the spatial domain \mathbf{D} of the transport problem (1) is divided into a number of subdomains \mathbf{D}_l ($\mathbf{D} = \bigcup_l \mathbf{D}_l$). We apply spatial domain decomposition to the transport equation in a discretized form

$$\mathcal{L}_h\psi_{h,l} = \mathcal{S}_h\psi_{h,l} + \frac{1}{4\pi}q_h, \quad \mathbf{r}_h \in \mathbf{D}_l, \quad \text{for all } \boldsymbol{\Omega}_m, \tag{6a}$$

$$\psi_{h,l}|_{\mathbf{r}_h \in \partial\mathbf{D}_l} = \psi_{h,l^*}|_{\mathbf{r}_h \in \partial\mathbf{D}_l}, \quad \text{for } \boldsymbol{\Omega}_m \cdot \mathbf{e}_{n,l} < 0, \tag{6b}$$

$$l = 1, \dots, N_l,$$

where \mathcal{L}_h and \mathcal{S}_h are discretized operators \mathcal{L} and \mathcal{S} , respectively, $\psi_{h,l}$ is the vector of discrete angular flux in subdomain \mathbf{D}_l , \mathbf{r}_h defines the given spatial grid, $\boldsymbol{\Omega}_m$ is a discrete-ordinate direction, $\mathbf{e}_{n,l}$ is outward normal at the boundary of l -th subdomain, l^* is the index of neighboring subdomain. The transport problem (6) can be solved locally in each subdomain with a prescribed lagged angular fluxes coming from surrounding subdomains. The integral transport matrix method (ITMM) is a spatial domain decomposition method for solving the discretized particle transport equation [11–14]. It uses an integral form of Eqs. (6) and applies response-matrix approach [15,16] to formulate of the equations for the scalar flux in l -th subdomain

$$\phi_{h,l} = \mathcal{J}_{\phi,h}\phi_{h,l} + \mathcal{B}_{\phi,h}q_h + \mathcal{K}_{\phi,h}\psi_{h,l}^{in}, \quad \text{for } \mathbf{r}_h \in \mathbf{D}_l, \tag{7a}$$

and for the outgoing angular flux at the subdomain boundary

$$\psi_{h,l}^{out} = \mathcal{J}_{\psi,h}\phi_{h,l} + \mathcal{B}_{\psi,h}q_h + \mathcal{K}_{\psi,h}\psi_{h,l}^{in} \quad \text{for } \mathbf{r}_h \in \partial\mathbf{D}_l, \tag{7b}$$

where

$$\psi_{h,l}^{out} = \psi_{h,l}|_{\mathbf{r}_h \in \partial\mathbf{D}_l} \quad \text{for } \boldsymbol{\Omega}_m \cdot \mathbf{e}_{n,l} > 0, \tag{7c}$$

$$\psi_{h,l}^{in} = \psi_{h,l^*}^{out} = \psi_{h,l^*}|_{\mathbf{r}_h \in \partial\mathbf{D}_l} \quad \text{for } \boldsymbol{\Omega}_m \cdot \mathbf{e}_{n,l} < 0, \tag{7d}$$

$\mathcal{J}_{\alpha,h}$, $\mathcal{B}_{\alpha,h}$, and $\mathcal{K}_{\alpha,h}$ ($\alpha = \psi, \phi$) are corresponding discrete operators. The ITMM uses parallel block Jacobi (PBJ) algorithm to solve the transport problem in \mathbf{D} that is defined as follows:

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