Journal of Computational Physics 297 (2015) 584–598

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Boundary element based multiresolution shape optimisation in electrostatics

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ARTICLE INFO

Article history:
Received 7 February 2014
Received in revised form 4 December 2014
Accepted 13 May 2015
Available online 18 May 2015

Keywords:
Shape optimisation
Shape derivative
Boundary element method
Subdivision surfaces
Multiresolution analysis

ABSTRACT

We consider the shape optimisation of high-voltage devices subject to electrostatic field equations by combining fast boundary elements with multiresolution subdivision surfaces. The geometry of the domain is described with subdivision surfaces and different resolutions of the same geometry are used for optimisation and analysis. The primal and adjoint problems are discretised with the boundary element method using a sufficiently fine control mesh. For shape optimisation the geometry is updated starting from the coarsest control mesh with increasingly finer control meshes. The multiresolution approach effectively prevents the appearance of non-physical geometry oscillations in the optimised shapes. Moreover, there is no need for mesh regeneration or smoothing during the optimisation due to the absence of a volume mesh. We present several numerical experiments and one industrial application to demonstrate the robustness and versatility of the developed approach.

1. Introduction

The shape optimisation of high-voltage electrical devices, such as switchgear or transformers, serves as the driving application for our work. The prevention of electrical breakdown is one of the key considerations in the design of high-voltage devices [1,2]. In a first approximation, limiting the electric field strength on critical components can reduce a device's susceptibility to electric breakdown. The electric field strength is determined with the electrostatic field equations, which in absence of space charges reduce to the Laplace equation with Dirichlet boundary conditions [3]. By optimising the shape of critical components the maximum electric field strength on the surface, i.e., the normal flux, can often be considerably reduced. This may make it possible to shrink the size of a device and in turn lead to cost savings. In the approach introduced in this paper we systematically optimise the geometry of a device such that a cost functional consisting of the L_2-norm of the electric field strength is minimised.

The boundary element method (BEM) has clear advantages when applied to shape optimisation of high-voltage devices, see [4–8] for an introduction to BEM. First of all, BEM relies only on a surface discretisation so that there is no need to maintain an analysis-suitable volume discretisation during the shape optimisation process. Moreover, BEM is ideal for solving problems in unbounded domains that occur in electrostatic field analysis. In gradient-based shape optimisation the

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http://dx.doi.org/10.1016/j.jcp.2015.05.017

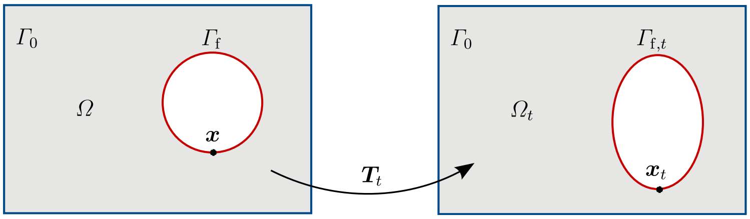

Fig. 1. Topology of the domain Ω and its transformation to the domain Ω_t.

shape derivative of the cost functional with respect to geometry perturbations is needed [9–11]. To this purpose, we use the adjoint approach and solve the primary and the adjoint boundary value problems with BEM. The associated linear systems of equations are dense and an acceleration technique, such as the fast multipole method [12,13], is necessary for their efficient solution. For some recent applications of fast BEM in shape optimisation and Bernoulli-type free-boundary problems we refer to [14–16].

The geometry parameterisation and its interplay with the BEM surface discretisation plays a crucial role in shape optimisation. When the BEM surface mesh is used for geometry parameterisation it leads to non-physical oscillations in the optimised geometry, as already known in the finite element literature [17,18]. In addition, the BEM mesh may become severely distorted after a few optimisation steps so that auxiliary mesh smoothing procedures become necessary. To remedy both difficulties, geometries in shape optimisation are commonly parameterised with b-splines or related techniques, such as NURBS and subdivision surfaces [17–21]. In this paper we represent geometries with subdivision surfaces, which are the generalisation of splines to arbitrary connectivity meshes. Specifically, we use the Loop scheme based on triangular meshes and quartic box-splines [22].

In subdivision schemes a limit surface is obtained through the repeated refinement of a coarse control mesh [23]. In practice, there are closed form expressions for computing the limit surface for a given control mesh [24,25]. The hierarchy of control meshes underlying a subdivision surface lends itself naturally to multiresolution editing [26,27]. The coarse control mesh vertex positions are modified to perform large-scale editing and the fine control mesh vertex positions are modified to add localised changes. In the introduced multiresolution shape optimisation approach we use a fine control mesh for BEM discretisation and coarser control meshes for geometry modification. More precisely, we start optimising with the coarsest control mesh and progress to optimise increasingly finer control meshes. As our numerical examples demonstrate, the multiresolution optimisation approach does not lead to non-physical oscillations in geometry. Moreover, the occurrence of mesh pathologies, like inverted elements, is greatly reduced because the support size of the geometry modifications and the element sizes are well coordinated.

This paper is organised as follows. In Section 2 we introduce the electrostatic shape optimisation problem and the required shape derivates. We then discuss in Section 3 the discretisation of the state and adjoint boundary value problems with the BEM. Subsequently, in Section 4 the multiresolution subdivision surfaces for geometry parameterisation are explained. The multiresolution optimisation algorithm is introduced in Section 5. Finally, in Section 6 we present several numerical examples with increasing complexity to demonstrate the efficiency and robustness of the proposed approach.

2. Electrostatic shape optimisation problem

The electrostatic field equations in absence of space charges lead to a Dirichlet boundary value problem for the Laplace equation

$$\begin{cases} -\Delta u = 0 & \text{in } \Omega, \\ u = 0 & \text{on } \Gamma_0, \\ u = 1 & \text{on } \Gamma_f, \end{cases} \tag{1}$$

where u is the electric potential or voltage, $\Omega \subset \mathbb{R}^3$ denotes a multiply connected, bounded Lipschitz domain with the boundary $\Gamma := \partial\Omega$ consisting of a free part Γ_f and a fixed part Γ_0. In this paper, we assume that the potentials on Γ_f and Γ_0 are constant. The geometry of the free boundary Γ_f is to be determined with shape optimisation. We assume that the topology of Ω is as shown in Fig. 1, i.e., that Γ_0 and Γ_f are disconnected parts of the boundary and that Γ_f is interior to Γ_0. It is straightforward to generalise our approach to other situations. Moreover, it is well known that the Dirichlet problem (1) admits a unique solution $u \in H^1(\Omega)$. For the purposes of the shape calculus introduced below, however, we assume a higher regularity of the solution u in the vicinity of the free part of the boundary Γ_f.

In electrostatic shape optimisation one may seek to minimise the pointwise maximum of the normal flux on the free part of the boundary Γ_f. The corresponding cost functional reads

$$J_{\max}(\Omega, u) := \sup_{\boldsymbol{x}\in\Gamma_f} \left| \frac{\partial u}{\partial \boldsymbol{n}}(\boldsymbol{x}) \right| \tag{2}$$