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Stable explicit coupling of the Yee scheme with a linear current model in fluctuating magnetized plasmas

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ABSTRACT

This work analyzes the stability of the Yee scheme for non-stationary Maxwell's equations coupled with a linear current model with density fluctuations. We show that the usual procedure may yield unstable scheme for physical situations that correspond to strongly magnetized plasmas in X-mode (TE) polarization. We propose to use first order clustered discretization of the vectorial product that gives back a stable coupling. We validate the schemes on some test cases representative of direct numerical simulations of X-mode in a magnetic fusion plasma including turbulence.

1. Introduction

The study of the impact of high turbulence level on the wave propagation introduces unexpected behavior on the stability of the numerical scheme, indeed numerical instabilities may arise after a very huge number of time-steps, as it will be shown. At the same time due to the high turbulence level, it becomes irrelevant to use the standard analysis using dispersion relation, that is the reason we propose to use another analysis based on energy to establish the stability criterion of the numerical scheme. A detailed study is provided to explain the way of thinking, and to facilitate the understanding of this work. Up to now, the role of turbulence on the wave propagation was ignored, but it becomes more and more important to explain experiments. The numerical modeling of these observed macroscopic phenomenons resulting from turbulent processes in magnetic plasmas requires huge numerical simulations. Such simulations should capture the statistical properties so that one can realize relevant averaging in time and space. This is particularly true to explain the macroscopic wave behavior which takes place on time scales that diagnostics cannot measure. An example which motivates our work is the recovery of the smooth envelope of a probing beam: to be able to compare with the theory, we need to simulate the non-stationary Maxwell equations with a linear current model, see Eqs. (1)–(2) below, in a domain with very fine non-homogeneities (since the underlying plasma is turbulent) and over a simulation time equivalent to a half-million wave periods. This is required to study in detail the role of the wavenumber spectrum on the beam widening [7,8,26], and is illustrated

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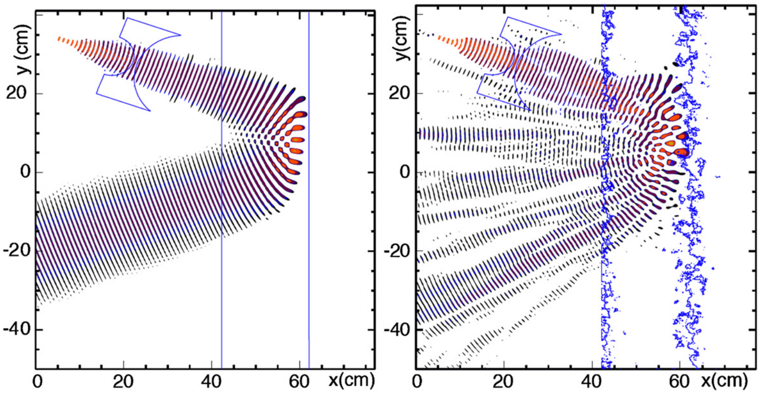

Fig. 1. Contours at a given time of the positive part of the electric field, that is $\max(E_z, 0)$, of an ordinary wave (known as TM or O-mode) with a Gaussian-like shape in unperturbed plasma with a linear density profile (left) and with homogeneous turbulence of $\delta n/n_c = 5\%$ (RMS value, with n_c the density at the cut-off layer for $\nu = 40$ GHz) where the sub-beams are present (right).

with a snapshot of a numerical calculation of a wave in a plasma in Fig. 1. The Gaussian shape is recovered at coarse scale using some time averaging, but at the same time very fine spatial scale details are visible. Since the energy of the wave is conserved during its propagation (we will prove this property for the model used in this work), it seems unavoidable to ask for similar energy preservation properties for the numerical methods in order to capture both the fine and coarse scales in the computations.

The previous example corresponding to beam broadening induced by wave propagation in turbulent plasma is illustrative of diagnostics and wave heating systems used in magnetized fusion plasmas. Hot topics for the ITER design concern the wavenumber resolution of the so-called Doppler reflectometry or on the choice of the probing frequency of the Coherent Thomson Scattering using X-mode as well as the beam widening induced by the turbulence on the electron cyclotron heating to determine whether neo-classical tearing modes inducing big islands can be controlled or not [21]. The same questions arise for the lower-hybrid heating system to evaluate correctly what is the wavenumber spectrum launched into the plasma in view of the efficiency prediction of the non-inductive current driven system and the definition of the energy deposition zone. In the near future, wave polarization changes will be a subject of great interest, as it is becoming with O–X mode heating scenario for the stellarator or tokamak [18], which are subject to anomalous reflection due to turbulence [22], and also for the new diagnostic concepts based on wave polarization changes induced by linear mode conversion or turbulence in inhomogeneous magnetized plasmas of space, astrophysics or fusion.

To our knowledge the state of the art of the computations done for such problems use simplified wave models taking into account only one physical mechanism (refraction in [21], diffraction effects in [31] and recently both in [16,17]) but not all the wave structure as a Maxwell's equation solver with a linear current model can predict. This strongly questions the numerical stability of the simulations, indeed our numerical tests show a fundamental stability issue on extraordinary mode (also called TE or X-mode) which are more demanding in terms of stability than O-mode computations. Because an ideal scheme should be fast enough to reach the requirements stressed above for the comparison between theory and full-wave simulations, we are strongly interested in explicit FDTD (Finite Difference Time Domain) schemes, indeed they are still the cheapest in terms of CPU. Many FDTD methods were developed to improve the performances and the stability in the past few years, such as EJ collocated FDTD [34], Runge–Kutta exponential time differencing formulation (RKETD) FDTD [20], matrix exponential (ME) FDTD [14], and exponential time differencing (ETD) FDTD [15]. Unfortunately they cannot be easily used to model the general dispersive media. On the other hand, in order to overcome the Courant–Friedrich–Levy (CFL) constraint of the conventional FDTD method, the one-step leapfrog ADI-FDTD method has been developed [4]. It originates from the conventional ADI-FDTD for which we refer to [35]. We finally mention the fact that conservation of energy for a fourth-order FDTD scheme is addressed in [19], and that massive direct simulations of wave propagation in a different physical context can be found in the recent references [12,10,23]. A recent variant of FDTD methods with higher computational efficiency is the implicit method [27] or [30,29], the latter being proved to be stable for varying coefficients. However ITER size simulations for the high frequency cases with small spatial scale turbulence that we consider in Fig. 1 seem not reachable using implicit schemes such as [30]. The reason is the very large number of cells (typically $\approx 10^{3d}$ with $d = 2, 3$ the dimension of the problem) which induces severe CPU constraints. This is why we study in this work the low cost explicit version of the Yee scheme coupled with a linear current. The stability of the method will be performed with an energy analysis which seems to us very convenient to handle strong gradients of the coefficients.

The general model problem considered in this work is the non-stationary Maxwell system

$$\begin{cases} -\varepsilon_0 \partial_t \mathbf{E} + \operatorname{curl} \mathbf{H} = \mathbf{J} \\ \mu_0 \partial_t \mathbf{H} + \operatorname{curl} \mathbf{E} = 0 \end{cases} \tag{1}$$

coupled with a linear equation for the electronic current density $\mathbf{J} = eN_e\mathbf{u}_e$,