Journal of Computational Physics 295 (2015) 87–113

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Strongly coupled dynamics of fluids and rigid-body systems with the immersed boundary projection method

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A R T I C L E   I N F O

Article history:
Received 15 September 2014
Received in revised form 20 March 2015
Accepted 8 April 2015
Available online 15 April 2015

Keywords:
Immersed boundary method
Fluid–structure interaction
Strong coupling
Relaxation

A B S T R A C T

A strong coupling algorithm is presented for simulating the dynamic interactions between incompressible viscous flows and rigid-body systems in both two- and three-dimensional problems. In this work, the Navier–Stokes equations for incompressible flow are solved on a uniform Cartesian grid by the vorticity-based immersed boundary projection method of Colonius and Taira. Dynamical equations for arbitrary rigid-body systems are also developed. The proposed coupling method attempts to unify the treatment of constraints in the fluid and structure—the incompressibility of the fluid, the linkages in the rigid-body system, and the conditions at the interface—through the use of Lagrange multipliers. The resulting partitioned system of equations is solved with a simple relaxation scheme, based on an identification of virtual inertia from the fluid. The scheme achieves convergence in only 2 to 5 iterations per time step for a wide variety of mass ratios. The formulation requires that only a subset of the discrete fluid equations be solved in each iteration. Several two- and three-dimensional numerical tests are conducted to validate and demonstrate the method, including a falling cylinder, flapping of flexible wings, self-excited oscillations of a system of many linked plates in a free stream, and passive pivoting of a finite aspect ratio plate under the influence of gravity in a free stream. The results from the current method are compared with previous experimental and numerical results and good agreement is achieved.

1. Introduction

Fluid–structure interaction (FSI) problems arise frequently in many scientific and engineering disciplines. These problems can be broadly defined as those in which a fluid interacts dynamically with a solid structure, in contrast to interactions in which the structure's kinematics are prescribed. In this work, we are primarily interested in FSI problems in which objects or structures undergo large scale motion or deformation in response to fluid forcing at moderate Reynolds numbers. Examples of such interactions arise in a variety of contexts: in biological locomotion, particularly exemplified by swimming of aquatic organisms or flight with flexible wings; in biomedical flows, such as those in the cardiovascular or pulmonary systems; and in transport of passive (or active) particles, as in suspensions or sedimentary flows. A general feature of such interactions is that the governing equations of the fluid and the structure are coupled in a highly non-linear manner. In this paper, we focus on structures composed of one or more rigid bodies, possibly linked into extended structures, that interact

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http://dx.doi.org/10.1016/j.jcp.2015.04.005

with incompressible flows. Though this narrower focus restricts the class of FSI problems amenable to the methodology outlined in this paper, a rich set of problems nonetheless remains.

Numerical simulations are widely employed to investigate such FSI problems. General discussions of computational FSI are provided in some recent reviews, e.g. [18,7]. The general task of an FSI methodology is to ensure that the kinematic and dynamic properties match at the interface between the fluid and the structure. When the constituent fluid and structure solvers are based on conforming mesh methods, in which the computational mesh of each material is aligned with the interface, then these properties are generally matched explicitly. Arbitrary Lagrangian Eulerian (ALE) methods are typically used to locally update the fluid mesh in response to advances in the structure configuration. (See, for example, the ALE-based coupling procedure developed by Farhat et al. [11] for treating aero-elasticity problems.)

Alternatively, one could view the task at the interface in the sense of variational mechanics, that FSI requires enforcement of the kinematic constraint (no-slip, no-penetration) at the interface, and that the associated interfacial force is the Lagrange multiplier that enforces this constraint. This latter perspective is particularly attractive as it unifies the interpretation of interfacial constraints with that of other constraints in the problem: the volume-preserving constraint in the incompressible fluid, for which the pressure acts as the Lagrange multiplier; and linkage constraints in the rigid-body systems, in which constraint forces serve the role of Lagrange multipliers. (Strictly speaking, one could also enforce the rigidity of the bodies themselves with Lagrange multipliers, but such a constraint is more naturally enforced directly by solving the rigid-body equations of motion.) This perspective is valuable when the interface conditions cannot easily be enforced directly, as when the fluid equations are solved with non-conforming mesh methods (sometimes referred to as immersed boundary, Cartesian grid or embedded boundary methods) [29]. For example, this is the underlying principle of the fictitious domain method developed by Glowinski and co-workers [15,14], in which Lagrange multipliers are distributed throughout the body interiors within a finite element formulation on a fixed mesh, thereby forcing the (fictitious) fluid in these interiors to move as rigid bodies.

The present work adopts the same perspective, but with notable differences in implementation compared with the fictitious domain methods. The framework presented here is based on the immersed boundary projection method, developed by Colonius and Taira [35,6] for flows around bodies with prescribed kinematics. In their finite volume (or staggered finite difference) method—inspired in part by the original immersed boundary method of Peskin [31,32]—Lagrange multipliers are distributed only on the fluid–structure interface and interpolated to a uniform Cartesian grid with discrete delta functions. The discrete system of equations takes the form of a saddle-point problem [3], in which the off-diagonal blocks are associated with the interface and incompressibility constraints. Here, we replace the prescribed body motions with the dynamical equations for the rigid-body system, which themselves possess a saddle-point form by virtue of the linkage constraints. The role of the Lagrange multipliers on the interface is unchanged, but now their effect is accounted for in both the fluid and the structure equations. The resulting global system of equations is differential-algebraic (time-discrete in the fluid but continuous in the structure) and retains an overall saddle-point form.

Because the assembled system equations is still of a general saddle-point form, one could, in principle, use any of several standard techniques—such as block Gauss elimination—to solve for the discrete fluid velocity field and body accelerations. However, in order to provide more freedom and avoid the poor conditioning of heterogeneous matrix blocks, the system is partitioned into separate fluid and structure blocks, each of which is solved by a null-space projection approach [5,6,33] to enforce the respective constraints of incompressibility and rigid-body linkages; in the fluid, this is equivalent to a vorticity-based treatment [5,6].

The coupling of the two solvers is of particular importance in the algorithm design, and partitioned numerical methods for FSI can be classified into two categories in this respect: weakly coupled and strongly coupled. In a weakly coupled algorithm, the kinematic and dynamic conditions are not enforced simultaneously. Rather, the solvers are advanced sequentially, wherein one solver—usually the structure solver—provides the other with kinematic conditions at the interface, and this solver is advanced in turn to compute interfacial tractions. This approach provides the advantages of simplicity—particularly in cases in which the partitioned solvers have been developed separately—and fixed computational load, since no iteration is pursued. However, the stability of these methods is not guaranteed and is often difficult to achieve for incompressible flows with highly flexible structures [4,12]. Therefore, weakly-coupled algorithms are widely employed in aero-elasticity problems, in which structural deformations are modest, but has also been used with some success in biomedical applications (e.g. [25]).

In a strongly-coupled partitioned scheme, as we pursue here, the kinematic and dynamic conditions are enforced simultaneously at the fluid–structure interface. The stability of this kind of scheme is greatly improved compared with a weak one, especially with structures undergoing large deformation. As a trade-off, the computational load becomes higher as iterations are generally required in every time step. A variety of iterative techniques have been introduced and employed in different types of fluid–structure systems [7]. Among these is the block Newton–Raphson approach, which generally converges rapidly but requires the computationally-expensive calculation of the Jacobian [27,26,8]. Alternatively, one may use a block Gauss–Seidel method (also known as fixed-point iteration), in which each solver is provided with the updated data from the other in every iteration, and the system is iterated to convergence within some tolerance. In its naive form, this method often converges slowly, particularly when the densities of the fluid and structure are quite disparate. However, this convergence can be improved with some degree of relaxation, as in [23] and the recent immersed-boundary FSI of Tian et al. [36], or by including some information about the virtual inertia (i.e. added mass) of the fluid in the structural solver [4,10].