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A simple, efficient, and high-order accurate curved sliding-mesh interface approach to spectral difference method on coupled rotating and stationary domains

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ABSTRACT

This paper presents a simple, efficient, and high-order accurate sliding-mesh interface approach to the spectral difference (SD) method. We demonstrate the approach by solving the two-dimensional compressible Navier–Stokes equations on quadrilateral grids. This approach is an extension of the straight mortar method originally designed for stationary domains [7,8]. Our sliding method creates curved dynamic mortars on sliding-mesh interfaces to couple rotating and stationary domains. On the nonconforming sliding-mesh interfaces, the related variables are first projected from cell faces to mortars to compute common fluxes, and then the common fluxes are projected back from the mortars to the cell faces to ensure conservation. To verify the spatial order of accuracy of the sliding-mesh spectral difference (SSD) method, both inviscid and viscous flow cases are tested. It is shown that the SSD method preserves the high-order accuracy of the SD method. Meanwhile, the SSD method is found to be very efficient in terms of computational cost. This novel sliding-mesh interface method is very suitable for parallel processing with domain decomposition. It can be applied to a wide range of problems, such as the hydrodynamics of marine propellers, the aerodynamics of rotorcraft, wind turbines, and oscillating wing power generators, etc.

1. Introduction

High-order (third and above) numerical methods are becoming more and more popular in recent years due to their capability of producing more accurate solutions on relatively coarse grid [31]. The spectral difference (SD) method is one discontinuous high-order method for solving the conservation laws on unstructured grids [15,32,28,11]. This method is an extension of the staggered multi-domain high-order method originally designed by Kopriva and Kolias [9]. It was shown that the SD method also has strong connection with the Flux Reconstruction/Correction Procedure via Reconstruction (FR/CPR) methods [5], and it shares similarity with the quadrature-free discontinuous Galerkin method [19]. The stability of a particular choice of flux points for the SD method was proved by Jameson [6] for the one-dimensional linear wave equation. Although the proof has not been generalized to higher-dimensional tensor-product elements, we have not observed numerical instability from several successful turbulent flow simulations [14,1,22].

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We have seen more and more applications of the SD method to realistic flow simulations, for example, for large eddy simulations on fixed grids [14,1,22,25,24,16]. The SD method is also particularly promising for simulating vortex-dominated flows on moving and deforming grids [23,34]. Liang et al. [12] extended the SD method for simulating two-dimensional unsteady flows around a plunging or pitching airfoil. DeJong and Liang [2] studied three-dimensional vortex induced vibrations using the SD method.

However, when the mesh undergoes very large rotation motion, such as for flows around rotating propellers or passing a flapping wing with very large pitching angles, remeshing [29,30] is required. Our goal is to involve the minimum number of remeshing and simultaneously preserve the high-order accuracy of the SD method. This motivates us to develop a new approach to the SD method for coupled rotating and stationary domains with sliding-mesh interfaces. In our approach, both inviscid and viscous fluxes on the sliding-mesh interfaces are constructed using a newly developed curved dynamic mortar method. The mortar method on fixed grids was originally proposed for incompressible flows by Mavriplis [18]. Kopriva [7,8] proved the conservation property of the mortar method for the compressible flow equations and applied it to the compressible Euler and Navier–Stokes equations on stationary domains using structured grids. In this paper, we show that our sliding-mesh approach is as simple as those designed for low-order numerical methods [33,20] while preserving the high-order accuracy of the SD method. This simple but novel sliding-mesh spectral difference (SSD) method can have a wide range of applications, such as marine propulsor hydrodynamics, rotorcraft aerodynamics, wind turbine wake dynamics, and oscillating wing power generators.

The paper is organized as follows: Section 2 gives the two-dimensional compressible Navier–Stokes equations on stationary and rotating domains. Section 3 reviews the SD method and presents the SSD method in detail. Verification studies and applications are reported in Section 4. Section 5 concludes the paper.

2. The governing equations

2.1. The compressible Navier–Stokes equations on stationary domain

We consider the two-dimensional unsteady compressible Navier–Stokes equations in conservative form,

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F}}{\partial x} + \frac{\partial \mathbf{G}}{\partial y} = 0, \tag{1}$$

where $\mathbf{Q}$ is the vector of conservative variables, $\mathbf{F}$ and $\mathbf{G}$ are the x and the y flux vectors. These terms have the following expressions,

$$\mathbf{Q} = [\rho \ \rho u \ \rho v \ E]^T, \tag{2}$$

$$\mathbf{F} = \mathbf{F}_{inv}(Q) + \mathbf{F}_{vis}(Q, \nabla Q), \tag{3}$$

$$\mathbf{G} = \mathbf{G}_{inv}(Q) + \mathbf{G}_{vis}(Q, \nabla Q), \tag{4}$$

where ρ is the fluid density, u and v are the x and the y velocities, E is the total energy per volume defined as $E = p/(\gamma - 1) + \frac{1}{2}\rho(u^2 + v^2)$, p is the pressure, γ is the ratio of specific heats and is set to 1.4 (i.e., the typical value for the air in standard conditions).

As shown in Eqs. (3) and (4), the fluxes have been divided into inviscid and viscous parts. The inviscid fluxes are only functions of conservative variables, which are

$$\mathbf{F}_{inv} = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho u v \\ (E + p)u \end{bmatrix}, \ \mathbf{G}_{inv} = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + p \\ (E + p)v \end{bmatrix}. \tag{5}$$

The viscous fluxes are functions of the conservative variables as well as their gradients. They have the following expressions,

$$\mathbf{F}_{vis} = -\begin{bmatrix} 0 \\ \tau_{xx} \\ \tau_{yx} \\ u\tau_{xx} + v\tau_{yx} + \mathrm{k}T_x \end{bmatrix}, \ \mathbf{G}_{vis} = -\begin{bmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ u\tau_{xy} + v\tau_{yy} + \mathrm{k}T_y \end{bmatrix}, \tag{6}$$

where τ_{ij} is the shear stress tensor and is related to the velocity gradients as $\tau_{ij} = \mu(u_{i,j} + u_{j,i}) + \lambda\delta_{ij}u_{k,k}$, μ is the dynamic viscosity, $\lambda = -2/3\mu$ based on Stokes' hypothesis, δ_{ij} is the Kronecker delta, k is the thermal conductivity, T is the temperature which is related to density and pressure through the ideal gas law $p = \rho RT$, where R is the gas constant.

2.2. The compressible Navier–Stokes equations on rotating domain

On the rotating domains, we implement a simplified equation which is equivalent to the Arbitrary Lagrange–Eulerian (ALE) [4] form of Eq. (1). Due to grid motion, the inviscid fluxes are modified to take the following forms,