Journal of Computational Physics 295 (2015) 285–306

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Hybrid spectral difference/embedded finite volume method for conservation laws

Jung J. Choi

The Department of Aerospace Engineering, University of Maryland, College Park, MD 20742, USA

ARTICLE INFO

Article history:
Received 25 September 2014
Received in revised form 2 March 2015
Accepted 10 April 2015
Available online 22 April 2015

Keywords:
Shock–turbulence interaction
Hybrid method
Spectral difference method
Finite volume method
Shock-capturing scheme

ABSTRACT

Recently, interests have been increasing towards applying the high-order methods to various engineering applications with complex geometries [30]. As a result, a family of discontinuous high-order methods, such as Discontinuous Galerkin (DG), Spectral Volume (SV) and Spectral Difference (SD) methods, is under active development. These methods provide high-order accurate solutions and are highly parallelizable due to the local solution reconstruction within each element. But, these methods suffer from the Gibbs phenomena when discontinuities are present in the flow fields. Various types of limiters [43–45] and artificial viscosity [46,48] have been employed to overcome this problem.

A novel hybrid spectral difference/embedded finite volume method is introduced in order to apply a discontinuous high-order method for large scale engineering applications involving discontinuities in the flows with complex geometries. In the proposed hybrid approach, the finite volume (FV) element, consisting of structured FV subcells, is embedded in the base hexahedral element containing discontinuity, and an FV based high-order shock-capturing scheme is employed to overcome the Gibbs phenomena. Thus, a discontinuity is captured at the resolution of FV subcells within an embedded FV element. In the smooth flow region, the SD element is used in the base hexahedral element. Then, the governing equations are solved by the SD method. The SD method is chosen for its low numerical dissipation and computational efficiency preserving high-order accurate solutions. The coupling between the SD element and the FV element is achieved by the globally conserved mortar method [56]. In this paper, the 5th-order WENO scheme with the characteristic decomposition is employed as the shock-capturing scheme in the embedded FV element, and the 5th-order SD method is used in the smooth flow field.

The order of accuracy study and various 1D and 2D test cases are carried out, which involve the discontinuities and vortex flows. Overall, it is shown that the proposed hybrid method results in comparable or better simulation results compared with the standalone WENO scheme when the same number of solution DOF is considered in both SD and FV elements.

1. Introduction

The interaction of turbulence and discontinuities, e.g. shock, detonation and contact surface, in the high speed flows is commonly encountered in many engineering applications. Turbulence interacting with shock formed by a high speed aircraft generates noise downstream of the shock, which travels to the ground and is called the sonic boom. In such case,

E-mail address: jungchoi@umd.edu.

http://dx.doi.org/10.1016/j.jcp.2015.04.013
0021-9991/

large scale turbulent motion interacting with shock can intensify or thicken shock, which in turn affects the level of noise downstream of the shock. Fundamental study of shock/turbulence interaction to understand the physics of the sonic boom and its reduction has been given a great attention theoretically and numerically [1,2]. The shock/shear layer interaction in the high speed turbulent jet from the aircraft engine also produces screech noise, and the reduction of such noise without compensating engine performance is of great interests [3]. For high speed propulsion systems, such as ramjet/scramjet, shock interacting with turbulent flows at high speed can enhance fuel/air mixing for a stable combustion with the aid of flame holding apparatus [4]. The interaction of shock and turbulent flame plays a crucial role in the deflagration to detonation transition [5].

Turbulence is a vortex dominated chaotic process with a wide range of spatial and temporal scales while shock is in the length scale of the molecular mean free path and is approximated by a mathematical discontinuity. These physical processes occurring at different length scales pose conflicting numerical requirements for successful simulations. In order to capture a wide range of length scales in turbulent flows, a numerical scheme with low numerical dissipation is required especially to capture the high wave number spectrum. On the other hand, the introduction of some numerical dissipation is needed for the numerical stability to capture discontinuities in the flow fields on a computational grid appropriate for resolving turbulent length scale. Therefore, devising a stable and accurate numerical method for simulating turbulence interacting with discontinuities in the flow fields is a challenging task.

Various low-dissipation high-order methods have been extensively developed to reduce the numerical dissipation in the smooth or turbulent flow fields yet provide the discontinuity-capturing capability with some numerical dissipation introduced in the vicinity of discontinuity. Among various high-order schemes, weighted essentially non-oscillatory (WENO) type scheme [6,7] and spectral-like compact scheme [8] are widely employed. The WENO family schemes are mainly shock-capturing schemes where the order of accuracy is reduced at the location of discontinuity while the order of accuracy is preserved in the smooth flow field. However, it has been noted that the WENO scheme becomes dissipative in the smooth or turbulent flow. The compact scheme is designed to have very low numerical dissipation resulting in spectral-like accuracy in the smooth or turbulent flow. But the compact scheme suffers from the Gibbs phenomena when discontinuity is present in the flow field. Various efforts have been made to address these issues associated with the WENO and the compact schemes since they are introduced, especially in applying these schemes for large eddy simulation (LES) and direct numerical simulation (DNS).

With the success of early WENO scheme, a lot of efforts have been made in order to lower the numerical dissipation in the smooth or turbulent flows. Balsara and Shu [9] noted that the WENO scheme is not monotonicity preserving and, thus, introduced a monotonicity preserving weighted essentially non-oscillatory (MPWENO) scheme. They employed the monotonicity preserving bounds of Suresh and Hyunh [10] and showed that the 9th or higher order MPWENO scheme has high phase accuracy, thus suitable for compressible turbulent flows. Martin et al. [12] introduced a bandwidth-optimized WENO scheme for the direct numerical simulation of turbulent compressible flows. They proposed a set of candidate stencils to be symmetric with an additional candidate stencil. Then, the bandwidth optimized weights for the optimal stencil are computed by minimizing the truncation error on a given grid to maintain a small dissipation error at high wavenumbers. The simulations of incompressible and high turbulent Mach number isotropic turbulence show good agreement compared with the simulations by the 6th-order central Pade scheme and show good high wavenumber characteristics of turbulent compressible flows. For reviews of ENO and WENO schemes and other variants, see Refs. [7,13,14].

The compact scheme provides spectral-like accuracy with very low numerical dissipation, but it is shown that the compact scheme suffers from the Gibbs phenomena when discontinuities are present. In order to remedy this problem, artificial viscosity/diffusivity, filter scheme and TVD limiter are utilized in the compact scheme for discontinuities in the flow fields. Cook and Cabot [15,16] introduced a high wavenumber biased spectral-like artificial shear and bulk viscosity based on the strain rate for discontinuity capturing in the compact scheme. Considering supersonic reacting flows with discontinuities, Fiorina and Lele [17] added artificial diffusivities to energy and species equations in addition to the artificial viscosity introduced in Cook and Cabot [16], and Kawai and Lele [18] extended this approach for curvilinear meshes. Mani et al. [19] noted that the artificial bulk viscosity in Cook and Cabot [16] significantly damps out the sound field while the turbulent field is not affected. Therefore, Mani et al. proposed to replace the strain rate by the dilatation and multiply it by the Heaviside function to localize the artificial bulk viscosity near the shock and showed the improvement in sound field prediction. Yee et al. [20] employed the artificial compression method (ACM) switch [22] as a characteristic filter to stabilize the numerical solutions and minimize the numerical dissipation near the discontinuities for the compact scheme. By extending this filter approach, Yee and Sjögreen [21] further developed the high order filter scheme for multiscale Navier–Stokes and magnetohydrodynamics (MHD) systems. The alternative approach was proposed by Cockburn and Shu [23] by using the TVD limiter and was modified by Yee [24] reducing spurious oscillations near the discontinuities which originate from the TVD limiter.

Another class of favored approach is the hybrid method [25]. The hybrid method combines a high-order low-dissipative method in the smooth or turbulent flow fields and a high-order shock-capturing scheme localizing the numerical dissipation in the proximity of discontinuities. In order to switch between two schemes, a discontinuity detector is devised. Adams and Shariff [26] introduced a hybrid method combining nonconservative compact scheme and ENO scheme. Following Adams and Shariff, Pirozzoli [27] used the WENO scheme in the region with discontinuities while the conservative compact scheme is used in the smooth flow region. Hill and Pullin [28] proposed a tuned hybrid center-difference/WENO method by developing a tuned centered difference scheme for the bandwidth optimization and coupling it with the WENO scheme for large eddy simulations with strong shocks. In these hybrid methods, it is noted that the numerical scheme to