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Low Mach number algorithm for droplet-laden turbulent channel flow including phase transition

A. Bukhvostova [a,*], J.G.M. Kuerten [a,b], B.J. Geurts [a,c]

[a] Multiscale Modeling and Simulation, University of Twente, Faculty EEMCS, PO Box 217, 7500 AE Enschede, The Netherlands
[b] Eindhoven University of Technology, Department of Mechanical Engineering, PO Box 513, 5600 MB Eindhoven, The Netherlands
[c] Fluid Mechanics Laboratory, Eindhoven University of Technology, Department of Technical Physics, PO Box 513, 5600 MB Eindhoven, The Netherlands

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ABSTRACT

In this study we propose a new numerical algorithm for droplet-laden turbulent channel flow with phase transitions at low Mach numbers. The carrier gas is treated as compressible flow. In order to avoid very small time steps at low Mach numbers that would arise from stability requirements associated with explicit time-stepping we propose a new semi-explicit time integration method, applied to the low Mach number compressible flow equations. We perform a perturbation analysis in powers of the Mach number of the system of governing equations. The obtained decomposition of pressure into a space-independent part and a hydrodynamic part permits to apply a special pressure-based time integration algorithm for compressible flows at low Mach numbers. An important feature of the new numerical approach is the independence of the maximum allowed time step on the Mach number. In this study we validate the new method by comparing it with a fully explicit code for compressible flow at general Mach numbers showing a good agreement in all quantities of interest. The differences between the results of the two codes are on the order of the square of the Mach number caused by the disregard of high-order terms in the Mach number in the new algorithm. The relative difference found for a specific low value of the Mach number of 0.05 is on the order of 1% for instantaneous and mean quantities of the two phases. We also quantify the efficiency of the new algorithm by comparing the computational time it takes to simulate one time unit with both codes.

1. Introduction

Multiphase flows with a large number of droplets dispersed into a gas play an important role in a variety of technological applications and environmental problems. Examples include thermal processing in food manufacturing, air pollution control and heat transfer in power stations [1]. In this paper we investigate a coupled Euler–Lagrange model to simulate droplet-laden turbulent channel flow in which phase transition plays an important role. The first such Euler–Lagrange study of mass and heat transfer in droplet-laden turbulent flow was done by Mashayek in 1998 [2]. He conducted a simulation study, investigating homogeneous turbulence with two-way coupling between the gas and the dispersed droplet phase involving momentum, mass and energy of the system. Later, a study dedicated to the mixing layer with embedded evaporating

* Corresponding author. Tel.: +31 53 489 3383.
E-mail address: a.bukhvostova@utwente.nl (A. Bukhvostova).

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droplets was conducted by Miller and Bellan [3]. In [4] a cloud of inertial evaporating droplets, interacting with a non-isothermal droplet-laden turbulent planar jet, is considered. In the present paper we present a new time-stepping algorithm tailored to compressible flow at low Mach numbers and extend the work of Mashayek [2] to wall-bounded turbulence by investigating turbulent channel flow with a dispersed droplet phase undergoing phase transition. The models adopted here are identical to the models used in [3] and [4]. The main difference between the current study and the studies mentioned above is that we consider conditions in which not only evaporation of droplets but also their growth by condensation of the vapor phase are important.

In this paper we consider fully developed turbulent channel flow of a mixture of air and water vapor in which water droplets are dispersed. Fully developed turbulent channel flow is treated as homogeneous in the streamwise and spanwise directions [5] and therefore, periodic boundary conditions are used in these directions, as in [6].

A turbulent flow can be modeled in various ways. In this paper we make use of direct numerical simulation in which all scales of the flow, except the detailed flow around each droplet, are resolved. The solution is time dependent and the time step is limited by both stability and accuracy considerations.

In this study the mixture of air and water vapor will be referred to as the carrier phase or the carrier gas and the droplets as the dispersed phase. Incorporation of evaporation and condensation of the dispersed phase raises the question whether or not to include explicit compressibility of the carrier phase. If the carrier gas is assumed to be strictly incompressible then the inclusion of evaporation and condensation is subject to the condition that all instantaneous changes in the local mass density of air and water vapor cancel each other precisely throughout the domain. A full simulation model can be developed for such an incompressible carrier phase [7]. The incompressible treatment of the carrier phase would still permit to incorporate changes in mass density into the model using the equation of state and the Boussinesq approximation, which implies the dependence of mass density on temperature and vapor mass fraction. In our problem we relax the physical approximations further and allow the divergence of velocity not to be equal to zero as a result of phase transitions. This motivates us to treat the carrier phase as compressible flow which is characterized by a low Mach number, much smaller than 1 for the chosen initial conditions. The focus in this paper is on the development of a new low Mach number algorithm suitable for multiphase flow with phase transition, extending earlier work by Bell et al. [8]. This algorithm must be efficient for low Mach numbers: it should be possible to take a time step independent of Mach number as $\mathrm{Ma} \to 0$.

There are two main numerical techniques for the treatment of compressible low Mach number flows: density-based methods and pressure-based methods. Two types of time-marching procedures are applied in density-based methods: explicit and implicit algorithms. Explicit density-based methods have a stability restriction, called Courant–Friedrichs–Lewy (CFL) condition [9], which makes these algorithms computationally expensive at low Mach numbers. Our previous study of turbulent droplet-laden channel flow with phase transition was done with a density-based explicit time integration method [10], which was restricted to values of the Mach number higher than the realistic value of 0.005. In implicit density-based methods the system of governing equations for compressible turbulent flow is ill-conditioned making iterative solutions excessively time consuming [11]. In order to avoid this poor condition of the numerical problem two types of schemes are used in density-based methods: preconditioning [12] and asymptotic schemes [13]. The proposed techniques are only applicable to time-independent problems. In the present study all quantities are time dependent because of the turbulent flow and the phase transitions that occur.

In order to apply preconditioning to time-dependent problems the dual-time-stepping technique is normally used [14]. The dual time-step method could also be considered for the actual compressible equations, facing the challenge of resolving very fast acoustic signals at low Mach numbers. We selected a different path and focus on the low Mach number approximation obtained as a result of asymptotic analysis and some approximations. The resulting system of equations is treated with a hybrid time stepping method. The approximated system has as virtue the absence of large eigenvalues which allows to adopt most of the terms explicitly. We closely follow the work of Bell et al. (2004) [8] and extend this to multiphase problems.

Pressure-based methods are extensions of pressure correction methods used in incompressible flow [15,16]. In incompressible flow the pressure correction follows from a Poisson equation which is obtained from the condition that the velocity field has zero divergence. In case of compressible low Mach number flow this divergence-free constraint on the velocity field is not applicable. In order to obtain a Poisson equation for the pressure an expression for the divergence of velocity is derived employing the continuity equation and the equation of state [17,8].

To obtain a suitable time integration method for the problem of droplet-laden turbulent channel flow we first perform a perturbation analysis in powers of the Mach number of the governing system of equations. An asymptotic analysis of the Navier–Stokes equations for this regime of flow conditions was conducted before in the study by Zank and Matthaeus in 1991 [18]. They derived low Mach number equations from the compressible Navier–Stokes equations employing multiple time- and space-scale expansions in powers of the Mach number. The single time-scale and multiple space-scale analysis conducted by Klein in 1995 [19] gives insight into the low Mach number Euler equations. In this paper we use a multiple time scale, single space scale low Mach asymptotic analysis closely following Müller (1998) [13] and later work of Boger et al. (2012) [20]. This approach allows to distinguish advective and acoustic modes for turbulent flows at low Mach numbers [21]. We take into account the lowest-order terms in Mach number and get a simplified system of governing equations applicable to a compressible carrier phase at low Mach numbers. The specific feature of this new system is the decomposition of pressure into two parts, one of which is independent of the spatial coordinates. This part of pressure is connected with other thermodynamic quantities through the equation of state. Another part can be called the 'incompressible' pressure