Journal of Computational Physics 295 (2015) 438–455

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Simulation of elastic guided waves interacting with defects in arbitrarily long structures using the Scaled Boundary Finite Element Method

Hauke Gravenkamp [a,*], Carolin Birk [b], Chongmin Song [b]

[a] Federal Institute for Materials Research and Testing, 12200 Berlin, Germany
[b] School of Civil and Environmental Engineering, The University of New South Wales, Sydney, NSW 2052, Australia

ARTICLE INFO

Article history:
Received 20 December 2014
Received in revised form 17 April 2015
Accepted 20 April 2015
Available online 28 April 2015

Keywords:
Scaled Boundary Finite Element Method
Guided waves
Unbounded domains
Cracks

ABSTRACT

In this paper, an approach is presented to model the propagation of elastic waves and their interaction with defects in plate structures. The formulation is based on the Scaled Boundary Finite Element Method (SBFEM), a general semi-analytical method requiring the discretization of boundaries only. For a homogeneous finite or infinite plate section, only the through-thickness direction of the plate is discretized. To describe a defect, the full boundary of a short plate section of irregular shape is discretized. High-order spectral elements are employed for the discretization. The formulation for infinite plates can model the transmission into an unbounded domain exactly. Results are compared with conventional Finite Element Analyses in both time domain and frequency domain. The presented approach allows for the simulation of complex reflection and scattering phenomena using a very small number of degrees of freedom while the mesh consists of one-dimensional elements only.

1. Introduction

In this contribution we will focus on problems of the type as sketched in Fig. 1: elastic waves propagating along thin structures and being scattered and reflected off defects. The distance between the point of excitation and the defect can be very large compared to the structure's thickness. Additionally, the distance between the defect and the rear end of the structure is often large as well so that it is desirable to describe the structure as an unbounded domain. This is a typical set-up often addressed in the theoretical or experimental investigation of guided waves.

Important applications of elastic guided waves, particularly in the ultrasonic frequency range, can be found in non-destructive testing [1–3] and structural health monitoring [4] as well as material characterization [5]. The necessity of numerical modeling in this context originates from the rather complicated propagation behavior of guided wave modes, the number of which increases with frequency. This leads to complex signals to be interpreted in practical applications, particularly if guided waves interact with defects or if inhomogeneities in the geometry or material parameters have to be considered.

Accurate numerical modeling in this field is still a very challenging task, even though established Finite Element procedures are generally capable of modeling wave propagation phenomena in solid media. The problem lies in the small

* Corresponding author. Tel.: +49 30 8104 4289.
E-mail address: hauke.gravenkamp@bam.de (H. Gravenkamp).

http://dx.doi.org/10.1016/j.jcp.2015.04.032
0021-9991/

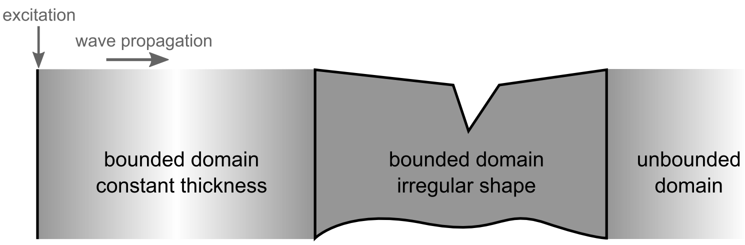

Fig. 1. Concept of modeling a plate containing a defect. In this example one bounded domain of irregular shape describes the defect. It is coupled to a regular bounded domain of constant thickness and an unbounded domain. The distance between the point of excitation and the defect can be large.

wavelengths of ultrasonic guided waves compared to the typically large dimensions of the structure to be modeled. This requires a very high number of degrees of freedom if the whole structure is discretized in a standard Finite Element or Finite Difference scheme. Additional difficulties, for instance stress singularities at crack tips or corners can further increase the number of degrees of freedom. At the same time, the high frequencies in the ultrasonic range require a small time increment in transient analyses. Typically, at least 10^3 to 10^4 time steps have to be considered to study simple reflection phenomena. The resulting computational times are often not acceptable for practical applications, especially if the simulations have to be performed many times in order to solve inverse problems. Higher-order element schemes are sometimes used in this context to enhance efficiency [6,7]. Additionally, applying the spectral cell method can reduce the meshing effort [8].

Since in many applications the waveguide contains long homogeneous sections of constant cross-section, a rather obvious idea to improve efficiency of the computation is to make use of the knowledge on wave propagation in these comparably simple systems. For plates and pipes of infinite length, analytical approaches are well-established to compute wavenumbers and mode shapes of guided wave modes (see e.g. [9–14]). The knowledge of the modal propagation behavior helps to analyze the signals or extract modal reflection coefficients from standard Finite Element analyses by means of modal decomposition. This approach has been used for instance to model the interaction of guided waves with notches [15–17] or cracks [18]. For certain applications, the size of the Finite Element models can drastically be reduced by applying the modal decomposition approach [19,20] or in a different manner by making use of orthogonality relations [21,22].

For more general geometries or inhomogeneous materials, the modal behavior in infinite waveguides can be obtained by using semi-analytical techniques, i.e., discretizing the cross-section of the waveguide in the Finite Element sense and describing the direction of wave propagation analytically. To the authors' knowledge, the first formulations of this approach were published by Kausel et al. in the 1970s and 80s [23,24]. They derived a quadratic eigenvalue problem to compute wavenumbers in plates or soil layers by using linear interpolation in the thickness direction. This approach – known as Thin Layer Method (TLM) – has been widely used in the modeling of soil [25,26] and later has been extended to e.g. anisotropic plates [27] and even piezo-composite layers [28]. Within the framework of this method, stiffness elements for layered strata have been derived [29], which can be computed in an analytical manner since only linear interpolation is applied in each layer. Based on a similar formulation, an absorbing boundary has been derived [30] as an alternative to other absorbing (non-reflecting) boundary conditions e.g. [31–33]. Interesting numerical details are provided in [34,35].

In the context of ultrasonic guided waves in solids, the concept of discretizing the cross-section only is often referred to as Semi-Analytical Finite Element (SAFE) Method [36–38], which uses the same quadratic eigenvalue problem as the TLM. It has been extended by discretizing a two-dimensional cross-section with standard Finite Elements in order to describe more general three-dimensional waveguides. Again, modal decomposition can be applied to propagate a given signal along a section of the waveguide [39,40]. Quite recently, the SAFE method has been coupled with standard Finite Element codes to model the interaction of guided waves with defects [41,42] or plate edges [43], which still requires an explicit modal decomposition of the displacement field as well as a high number of degrees of freedom to model cracks in the FEM region. Few publications discuss approaches based on the Boundary Element Method (BEM) [44]. The BEM can be applied to obtain dispersion relations for a prismatic waveguide, which requires the solution of a non-linear eigenvalue problem. Also, bounded domains can be treated by discretizing the full boundary. To the authors' knowledge, no BEM based solution has been presented to model a finite section of a plate by discretizing the thickness only.

A somewhat different approach that is being used is referred to as Waveguide Finite Elements, where a representative section of the waveguide (often of unit length) is discretized, yielding nodes on the opposite cross-sections only [45–47]. This is sometimes achieved by discretizing the section with standard Finite Elements and eliminating internal nodes by means of dynamic condensation [48]. Numerical issues of the method are discussed in [49]. A different method that should be mentioned in this context is based on the idea of coupling FE models of substructures of (periodic) waveguides by means of Lagrange multipliers [50,51].

The approach presented in the current paper is based on the Scaled Boundary Finite Element Method (SBFEM) [52–55]. The SBFEM is generally a semi-analytical method that requires discretization of the boundary of a domain only and combines many advantages of Finite Elements and Boundary Elements [56]. Originally, this method evolved from concepts to model unbounded domains using Finite Elements [57,58], one of these concepts being the Thin Layer Method, while the SBFEM