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# Using hybrid implicit Monte Carlo diffusion to simulate gray radiation hydrodynamics

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ABSTRACT

This work describes how to couple a hybrid Implicit Monte Carlo Diffusion (HIMCD) method with a Lagrangian hydrodynamics code to evaluate the coupled radiation hydrodynamics equations. This HIMCD method dynamically applies Implicit Monte Carlo Diffusion (IMD) [1] to regions of a problem that are opaque and diffusive while applying standard Implicit Monte Carlo (IMC) [2] to regions where the diffusion approximation is invalid. We show that this method significantly improves the computational efficiency as compared to a standard IMC/Hydrodynamics solver, when optically thick diffusive material is present, while maintaining accuracy. Two test cases are used to demonstrate the accuracy and performance of HIMCD as compared to IMC and IMD. The first is the Lowrie semi-analytic diffusive shock [3]. The second is a simple test case where the source radiation streams through optically thin material and heats a thick diffusive region of material causing it to rapidly expand. We found that HIMCD proves to be accurate, robust, and computationally efficient for these test problems.



## 1. Introduction

Many radiation hydrodynamic problems, such as those found in the fields of astrophysics and inertial confinement fusion, are composed of strongly heterogeneous materials that can vary in optical thicknesses by many orders of magnitude. A Hybrid Implicit Monte Carlo Diffusion (HIMCD) solver is developed in this work that can accurately resolve the radiation hydrodynamics equations more efficiently than the standard Implicit Monte Carlo (IMC) [2] approach typically used on these types of problems.

IMC is a robust method that is used to evaluate the radiative transfer equation and the associated material energy balance equation. IMC relies on a first order expansion of the change in material temperature in time to create a set of equations that are evaluated using a Monte Carlo tracking algorithm. This approach creates an additional term in the transport equation, typically referred to as "effective scattering", which physically accounts for photons being absorbed and subsequently re-emitted by the material over a time step. Effective scattering can make Monte Carlo prohibitively expensive when opaque materials are present. As a result, many methods have been developed to accelerate the solution of these equations in opaque materials [1,4,5].

Asymptotic analysis has shown that the diffusion approximation can accurately approximate the radiative transport solution in a thick diffuse limit [6]. Implicit Monte Carlo Diffusion (IMD) [1] can be used to accurately simulate the solution to

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the radiative transfer equation in regions where this limit is valid at a reduced cost compared to IMC [1,7]. IMD evaluates a spatial discretized diffusion equation using a Monte Carlo technique. There is a similar method in the literature known as Discrete Diffusion Monte Carlo (DDMC) [8] and differs primarily in the way that the photons are tracked in time. IMD tracks photons discretely in time based on the temporal discretization, and DDMC tracks photons continuously in time [7]. This work focuses on IMD but the reader should note that much of what is discussed is similarly applicable to DDMC. Similarly, many of the tools developed to implement this work were originally developed for DDMC and will be referenced as such.

In this work we will describe how HIMCD can be coupled to a Lagrangian hydrodynamics code to accurately resolve the fluid frame radiation hydrodynamics equations. This approach details how to use operator splitting to evaluate the material motion effects on the radiation field. We show that this operator splitting approach accurately resolves the coupled solution in diffusive radiation problems such as the Lowrie radiation hydrodynamics shock problem [3]. We also show that the coupled HIMCD/Hydrodynamics code can accurately and efficiently, as compared to standard IMC, resolve problems that have optically thick and optically thin materials with a simple two material ablation problem.

### *1.1. Hybrid implicit Monte Carlo diffusion*

Significant improvements in problem solution times have been demonstrated for HIMCD and Hybrid IMC/DDMC in problems that contain both thick diffusion and transparent streaming media [1,5]. HIMCD accelerates the time to solution of an IMC radiative transfer calculation. This acceleration is achieved by applying the well known diffusion approximation, using IMD, in regions of a problem that are dominated by isotropic scattering making diffusion an accurate approximation to transport. In diffusive regions, IMC particles take many short paths that can dominate the solution time. The IMD method approximates many short IMC paths with a single, much larger, path [1].

IMD is an ideal choice to combine with IMC, because of some important shared characteristics. The biggest advantage is that IMD takes advantage of similar data types used in IMC, Monte Carlo particles, that can be advanced in a similar Monte Carlo transport routine. Interface conditions can also be seamlessly integrated during a simulation allowing Monte Carlo particles to move between IMC and IMD regions. Densmore et al. [9] have developed a rigorous interface condition that will preserve the analytic emissivity of a face that separates DDMC and IMC regions. We will apply this interface condition for the IMD–IMC interface in HIMCD. IMC and IMD both apply the Fleck and Cummings [2] first order expansion of the emission source term, which creates a semi-implicit set of equations, avoiding the need for a non-linear iteration. IMD has been demonstrated to treat continuous frequency distributions similarly to IMC [7]. More recently DDMC has been demonstrated to perform well in frequency dependent problems using frequency integrated groups [10].

To the authors knowledge neither IMD or DDMC have previously been applied to the radiation hydrodynamics problems, and the remainder of this work will outline how HIMCD can be applied to such problems.

### *1.2. Radiation hydrodynamics*

This work implements the “fluid-frame” semi-implicit Lagrangian-frequency-independent radiation hydrodynamics equations. This means that the reference frame is considered to share the local fluid velocity and the radiation quantities are defined in reference to it [11]. This work can be extended to frequency dependent problems as in the work of Densmore et al. [10]. The equations presented here are for a single material, which is all that is required for Lagrangian Hydrodynamics.

It is an open question as to whether the fluid-frame IMC and IMD equations are strictly conservative. To determine this it is necessary to transform the equations into the lab frame and prove that these transformed equations are conservative. Preliminary work presented by Morel [12] suggests that the fluid frame diffusion equation is only conservative in the equilibrium diffusion limit.

#### *1.2.1. The implicit Monte Carlo radiation hydrodynamics equation*

There are four different coupled non-linear equations that comprise the set of frequency independent radiation hydrodynamics equations [11]. Variable dependencies will be defined in the text; however, they will be left out of the main equations for brevity. The first of the four radiation hydrodynamics equations expresses the time evolution of mass:

$$\frac{D\rho}{Dt} + \rho\bar{\nabla}\cdot\bar{u} = 0 \tag{1}$$

where $\bar{u}(\bar{r},t)$ [cm/s] is the material velocity at position $\bar{r}$ [cm] and time $t$ [s], and $\rho(\bar{r},t)$ [g/cc] is the density of the material. Here we have written the equations in terms of Lagrangian derivative:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \bar{u}\cdot\nabla. \tag{2}$$

The second equation expresses the time evolution of the material kinetic energy:

$$\rho\frac{D\bar{u}}{Dt} + \bar{\nabla}p - \kappa\bar{F} = 0 \tag{3}$$