Journal of Computational Physics 291 (2015) 34–51

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Multi-scale modelling of flow in periodic solid structures through spatial averaging

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ARTICLE INFO

Article history:
Received 22 May 2014
Received in revised form 24 February 2015
Accepted 26 February 2015
Available online 12 March 2015

Keywords:
Closure
Cylinder
Multi-scale modelling
Periodic flow
Spatial averaging
Tube
Volume averaging

ABSTRACT

This paper presents spatially averaged Navier–Stokes equations for modelling macro-scale flow in devices with periodic solid structures such as fin and tube arrays. The properties of steady and unsteady periodically developed flow are investigated to assess different strategies for determining the closure terms in the macro-scale flow equations. It is shown that the spatial averaging technique requires an appropriate weighting function to ensure that the closure terms are spatially constant for periodically developed flow. Moreover, through an appropriate choice of the weighting function, the closure terms can be obtained by solving a local closure problem on a unit cell of the periodic structures. The theoretical framework of this paper is applied as multi-scale modelling technique for flow through a cylindrical tube array. This case study illustrates the advantages of the weighted spatial averaging technique over the volume averaging technique.

1. Introduction

In heat exchangers, heat sinks, fuel cell systems as well as chemical reactors and many other applications, examples of fluid flow and heat transfer through spatially periodic solid structures are omnipresent. In heat transfer devices, for instance, periodic fin surfaces are employed to enhance heat transfer. For design of these devices, a detailed analysis of the flow and temperature fields through direct numerical simulation is not feasible, because of the enormous computational cost associated with simulating all the detailed transport phenomena near the solid–fluid interface.

In order to reduce computational cost, macro-scale models based on the volume averaging technique (VAT) for porous media [1] have been applied for simulating the flow and heat transfer in devices with periodic solid structures. These macro-scale models describe the features of the volume-averaged flow field and temperature over the global device scale and omit the need of investigating the local flow details over the entire flow domain.

Kuwahara, Nakayama et al. [2] used the VAT framework to correlate the macro-scale pressure gradient and convective heat transfer coefficient for a uniform macro-scale flow through a bank of isothermal square rods. Their correlations are governed by the Darcy–Forchheimer law for porous media and take the effect of macro-scale flow direction, Reynolds number and porosity into account. The laminar flow and heat transfer in a periodic array of square rods was further investigated by Alshare et al. [3], who correlated the volume averaged velocity and pressure gradient in steady periodically developed flow via apparent permeabilities. The authors also examine the effect of unsteady flow on the ensemble averaged

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http://dx.doi.org/10.1016/j.jcp.2015.02.051
0021-9991/

Nomenclature

$\boldsymbol{b}'_{fs}$	weighted interfacial force defined in Eq. (16)	[N/m^3]
C	differentiability class	
d	cylinder/tube diameter	[m]
$\boldsymbol{e}$	unit vector	[–]
$\boldsymbol{f}$	gravitational body force distribution	[N/m^3]
$\boldsymbol{g}$	gravitational acceleration	[m/s^2]
$\boldsymbol{G}_m^{(n)}$	geometrical tensor defined in Eq. (40)	
$\boldsymbol{I}$	unit tensor	[–]
$\boldsymbol{l}^{(j)}$	lattice vector of unit cell	[m]
l_j	lattice size (length lattice vector) of unit cell	[m]
$\boldsymbol{K}_m$	weighted permeability tensor defined in Eq. (52)	[m^2]
ℓ	characteristic length for micro-scale	[m]
$\mathcal{L}$	characteristic length for macro-scale	[m]
$\boldsymbol{M}'$	weighted momentum dispersion source defined in Eq. (16)	[m/s^2]
m	normalized weighting function	[m^{-3}]
N	number of cylindrical tubes	[–]
p	pressure distribution as in Eq. (2)	[Pa]
Δp	pressure drop over cylinder array	[Pa]
$\boldsymbol{n}_{fs}$	normal vector associated with fluid–solid interface	[–]
$\boldsymbol{r}$	position vector (locating point within averaging domain)	[m]
r_m	characteristic size of the averaging domain	[m]
Re	Reynolds number	
s_0	position parameter first cylinder	[m]
s_N	position parameter last cylinder	[m]
s_x	horizontal cylinder spacing	[m]
s_y	vertical cylinder spacing	[m]
t	time	[s]
$\boldsymbol{u}$	velocity distribution as in Eq. (2)	[m/s]
$\boldsymbol{x}$	position vector (locating centroid of averaging domain)	[m]

Greek symbols

δ_{fs}	Dirac distribution associated with fluid–solid interface	[m^{-1}]
ϵ_f	volume averaged fluid indicator, porosity of solid structure	[–]
ϵ_{fm}	weighted spatially averaged fluid indicator, weighted porosity of solid structure	[–]
ϵ_{sm}	weighted spatially averaged solid indicator	[–]
$\boldsymbol{\phi}$	distribution of a tensor quantity as in Eq. (1)	
γ_f	fluid indicator defined in Eq. (5)	[–]
γ_s	solid indicator defined in Eq. (5)	[–]
Γ_{fs}	fluid–solid interface	
$\bar{\Gamma}_{fs}$	fluid–solid interface within averaging domain	
$\boldsymbol{\Lambda}_m$	weighted momentum coefficient tensor defined in Eq. (53)	[–]
μ	dynamic viscosity of fluid	[Pa s]
ρ_f	constant fluid density	[kg/m^3]
Ω	continuum/device domain	
Ω_{unit}	unit cell domain	
$\bar{\Omega}$	averaging domain	
$\boldsymbol{\tau}$	viscous stress tensor	

Subscripts

f	restricted to the fluid
ref	used as reference
s	restricted to the solid
unit	unit cell
x	vector component along the x-direction
xx	first diagonal tensor component
yy	second diagonal tensor component

Superscripts

$+$	dimensionless quantity
$\sim$	deviation part defined in Eq. (6)
T	transpose of tensor

pressure gradient and provide correlations to incorporate thermal dispersion. In subsequent work [4], Alshare et al. applied VAT to simulate the macro-scale velocity and temperature distributions in a serpentine heat exchanger, using the closure correlations from their previous work. Their study discusses in depth the accuracy of the VAT model in comparison to direct numerical simulation. Horvat and Catton [5] modelled unidirectional macro-scale flow and heat transfer through a heat sink consisting of pin–fins on a solid base plate. Their numerical VAT model contains two closure coefficients to include local transport phenomena at the pin–fin surface: the local drag and local heat transfer coefficient, which are obtained from experimental correlations. In a related study [6], Horvat and Mavko calculate the local drag and heat transfer coefficient for a pin–fin element via a separate model based on periodical temperature and flow equations. For both studies the VAT model predicted Nusselt number and drag coefficient of the whole pin–fin section in good agreement with experimental results. Hierarchical VAT models for turbulent flow and heat transfer in heat sinks with circular and square pin–fins were developed by Catton [7]. Recently, VAT models were also applied for turbulent flow in circular tube–fin heat exchangers [8] and heat sinks with elliptic scale-roughened surfaces [9].

Although VAT models have proven their merit for describing transport processes in certain types of disordered porous media as one encounters in soils and aquifers, the volume averaging technique does not guarantee adequate modelling of flow and heat transfer in devices with spatially periodic structures, such as fins or tubes. The reason is that the volume averaged macro-scale models for porous media, which neglect the spatial moments of the solid structure, loose accuracy for systems in which the solid–fluid interface is exactly spatially periodic [10]. Therefore, we propose an alternative multi-scale modelling approach for simulating flow through periodic solid structures. The approach is based on a weighted spatial