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Dynamic multi-source X-ray tomography using a spacetime level set method

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ABSTRACT

A novel variant of the level set method is introduced for dynamic X-ray tomography. The target is allowed to change in time while being imaged by one or several source–detector pairs at a relatively high frame-rate. The algorithmic approach is motivated by the results in [22], showing that the modified level set method can tolerate highly incomplete projection data in stationary tomography. Furthermore, defining the level set function in spacetime enforces temporal continuity in the dynamic tomography context considered here. The tomographic reconstruction is found as a minimizer of a nonlinear functional. The functional contains a regularization term penalizing the L^2 norms of up to n derivatives of the reconstruction. The case $n = 1$ is shown to be equivalent to a convex Tikhonov problem that has a unique minimizer. For $n \geq 2$ the existence of a minimizer is proved under certain assumptions on the signal-to-noise ratio and the size of the regularization parameter. Numerical examples with both simulated and measured dynamic X-ray data are included, and the proposed method is found to yield reconstructions superior to standard methods such as FBP or non-negativity constrained Tikhonov regularization and favorably comparable to those of total variation regularization. Furthermore, the methodology can be adapted to a wide range of measurement arrangements with one or more X-ray sources.

1. Introduction

We consider the use of X-ray tomography for imaging moving targets. The key difficulty is that while the source–detector pair of a CT device rotates between recording two projection images, the target has already changed. Periodic movement such as the beating of a heart can be satisfactorily imaged using gating [32,18]. However, problems remain with tomographic imaging of non-periodic changes such as the flow of contrast agent inside blood vessels in angiography.

This work is motivated by multi-source tomography setups such as described already in 1983 in [33]. Consider placing several X-ray sources to irradiate a moving target simultaneously, and a corresponding set of X-ray detectors with high framerate. See Fig. 2. There are no moving parts in this arrangement, reducing calibration issues and offering simplifications for engineering and manufacturing.

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The above tomographic sampling geometry is quite peculiar. Temporal resolution can be very high as detectors with framerate of 400 fps or more are available off-the-shelf. However, the number of projection directions is severely limited simply by spatial constraints in placing X-ray tubes and bulky detectors in three-dimensional space. The experimental setup in [33] featured 28 source–detector pairs, which is still very few from the tomographic resolution-theory point of view [25]. Keeping the number of source–detector low, preferably under ten, offers significant economical and engineering advantages but leads to a very ill-posed tomographic problem.

There are encouraging results in the literature of extremely sparse-angle *stationary* tomography [37,23,35,36,3,4,15,19,14], including approaches based on level set methods [40,11,30,22,39,21]. See the introduction of [14] for more references. The above studies suggest that even less than 10 projection directions can be enough for tomographic reconstruction if the incomplete measurement information is effectively complemented with *a priori* information.

In particular, the modified level set method approach introduced in [22] is known to suppress sparse-data and limited-angle artifacts very effectively. The modification is to model the X-ray attenuation coefficient inside the level set by the continuous level set function itself instead of a constant, as in the classical level set method [29,27,16,41,9,6,28,5,7,10,8]. See Fig. 3.

However, incorporating time in the reconstruction calls for a novel algorithmic approach. In [26], three of the present authors extended the modified level set method to the dynamic case using a spacetime approach. Temporal regularization, or the promotion of relatively slow changes of the target in time, is offered simply by the requirement that the level set function is continuous in spacetime.

The promising results in [26] are based on simulated data of a simple target only. The goal of this paper is to generalize and analyze the spacetime level-set method introduced in [26] and to test it numerically in $(2+1)$-dimensional cases using

1. simulated data of more complicated targets than those used in [26], and
2. measured X-ray data of a temporally changing object.

Let us describe the principle of the spacetime level set method in the $(2+1)$-dimensional setting. The X-ray attenuation is modelled by a function of the form $f(u)$, where $u = u(x, y, t)$ is a smooth function and $f : \mathbb{R} \to \mathbb{R}$ is given by

$$f(\tau) = \begin{cases} \tau, & \text{if } \tau \geq 0, \\ 0, & \text{if } \tau < 0. \end{cases} \tag{1}$$

This means that the attenuation is zero *outside the level set*, in other words in the set $\{(x, y, t) \,|\, u(x, y, t) < 0\}$. Further, the attenuation coincides with $u(x, y, t)$ *inside the level set*, or in technical terms in $\{(x, y, t) \,|\, u(x, y, t) > 0\}$.

We find the level set function u as a minimizer of the functional $F_n : H^n(\Omega) \to \mathbb{R}$,

$$F_n(u) = \|\mathcal{A} f(u) - m\|^2_{L^2(E)} + \alpha \sum_{1 \leq |\beta| \leq n} \|D^\beta u\|^2_{L^2(\Omega)}, \tag{2}$$

where $\mathcal{A}$ is an operator modeling 2D Radon transforms measured at several times, $\beta = (\beta_1, \beta_2, \beta_3)$ is a multi-index with $|\beta| = \beta_1 + \beta_2 + \beta_3$, and $\alpha > 0$ is a regularization parameter.

For the special case $n = 1$ studied also in [22,26] we prove that minimizing F_n is equivalent to non-negativity constrained Tikhonov regularization

$$\operatorname*{arg\,min}_{\substack{u \in H^1(\Omega) \\ u \geq 0}} \left\{ \|\mathcal{A}u - m\|^2_{L^2(E)} + \alpha \|\nabla u\|^2_{L^2(\Omega)} \right\},$$

which has a unique solution. This result gives new insight into the connection between level set methods and Tikhonov regularization. In particular, this explains our numerical observation that the level set function never attains very negative values when $n = 1$.

Furthermore, we analyse the proposed method in the cases $n \geq 2$. Despite the fact that the case $n = 1$ essentially reduces to a convex minimization problem, for $n \geq 2$ such result is not available and we have to solve the non-convex minimization problem $\arg\min_{u \in H^n(\Omega)} F_n(u)$. See Fig. 1 for an illustration of the non-convexity. Moreover, the functional F_n is not coercive in H^n. However, we are able to use a non-standard argument to show that F_n has at least one global minimizer. Our strategy of proof is based on requiring the regularization parameter to satisfy a bound involving the signal-to-noise ratio, which is a practically relevant quantity that can typically be estimated from measurements.

The new spacetime level-set method is defined in a general form and can be readily extended to other measurement geometries. One of the most interesting extensions is one source–detector pair imaging a moving object on a slowly rotating platform. The rotation would provide a complete collection of projection directions for a static target, but in the present context the object is allowed to change in time.

Dynamic X-ray tomography is not a new idea. Regular CT devices are used dynamically all the time: a search in a scholarly database using the search term “dynamic CT” yields more than two million hits. However, this is typically based on filtered back-projection (FBP) algorithms, requiring fine angular sampling and thus being applicable only to slow or periodic motion. While there are dedicated dynamic FBP variants [34,20] and deformation-specific approaches [12], tracking