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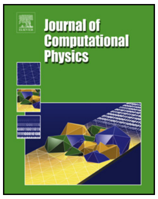

Topology preserving advection of implicit interfaces on Cartesian grids

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ABSTRACT

Accurate representation of implicit interface topology is important for the numerical computation of two phase flow on Cartesian grids. A new method is proposed for the construction of signed distance function by geometrically projecting interface topology onto the Cartesian grid using a multi-level projection framework. The method involves a stepwise improvement in the approximation to the signed distance function based on pointwise, piecewise and locally smooth reconstructions of the interface. We show that this approach provides accurate representation of the projected interface and its topology on the Cartesian grid, including the distance from the interface and the interface normal and curvature. The projected interface can be in the form of either a connected set of marker particles that evolve with Lagrangian advection, or a discrete set of points associated with an implicit interface that evolves with the advection of a scalar function. The signed distance function obtained with geometric projection is independent of the details of the scaler field, in contrast to the conventional approach where advection and reinitialization cannot be decoupled. As a result, errors introduced by reinitialization do not amplify advection errors, which leads to substantial improvement in both volume conservation and topology representation.

1. Introduction

Advection of implicit surfaces associated with scalar fields is widely used for propagating sharp interfaces on Cartesian grids [1]. This technique is relatively more effective in handling complex changes in interface topology [2] compared with the methods of Lagrangian interface tracking [3] and interface reconstruction from the volume-of-fluid elements [4]. The scalar fields however need to be reinitialized frequently to facilitate an accurate representation of the interface normal and curvature on the grid [5]. Reinitialization is also necessary for a uniformly smooth transition from one fluid to the other [6,7]. For sharp interface methods, the scalar field needs to be reinitialized to a signed distance function [8]. Diffused interface methods on the other hand attempt to maintain a transition zone by reinitializing the scalar field to an indicator function [9–12]. Moreover, a number of studies based on the sharp interface approach also find it convenient to employ a smooth transition of density or viscosity across the interface [13–15]. The accuracy of reinitialization of the scalar field is therefore necessary for a realistic representation of two phase flow on Cartesian grids.

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When the scalar field is represented by the level set function, reinitialization to a signed distance function can be implemented on the basis of an iterative solution of the unsteady, nonlinear Hamilton–Jacobi equation [7]. This procedure however perturbs the implicit interface away from its original position. The degree of perturbation depends on the difference between the scalar field and the corresponding signed distance function at the beginning of the reinitialization step. In order to maintain the scalar field as a signed distance function, reinitialization needs to be carried out frequently during advection. The errors introduced by reinitialization accumulate over time and amplify the advection errors, resulting in an overall loss of volume enclosed by the implicit interface [16]. To remedy the problem, Sussman and Fatemi [16] add a volume conservation constraint to help prevent the interface from drifting during reinitialization. Alternatively, Russo and Smereka [17] suggest that interfacial drift can be reduced by employing appropriate upwinding discretization of the Hamilton–Jacobi equation close to the interface. A more direct approach has been suggested by Enright et al. [18], where Lagrangian marker particles are employed to rebuild the signed distance function in under-resolved regions. Although the conservation of volume improves with the use of such techniques, the accuracy of representing interface topology on the Cartesian grid has not been investigated.

The implicit interface can also be defined by an indicator function that transitions smoothly from one fluid to the other over a transition region of finite width [9,19]. Here too, the numerical distortion of the indicator function during advection perturbs the interface. A reinitialization procedure is applied to correct the deviation by subjecting the indicator function to 'recompression' in directions normal to the interface [20]. With this approach, Olsson et al. [20] and Sheu et al. [10,21] report a significant improvement in volume conservation compared with reinitialization based on the Hamilton–Jacobi formulations. However, Desjardins et al. [14] note that even though the volume is conserved, the interface topology may still be degraded because of nonsmooth normals produced by recompression. The convergence behavior of interface normals and curvature has not been reported previously for this scheme. Moreover, the effectiveness of the recompression step depends on two adjustable parameters, i.e., a diffusion coefficient and an error tolerance, that are often problem dependent and may not be known a priori.

In this work we show that the lack of volume conservation and the distortion of the implicit interface during the advection of reinitialized scalar fields can be attributed to the coupled, nonlinear nature of these procedures. The advection of the implicit interface associated with reinitialized scalar fields, introduces additional errors that accumulate and amplify in time. Such errors can be minimized, in principle, by decoupling advection and reinitialization; leaving the implicit interface subject only to advection errors. This is not possible with the existing schemes, which cannot *reinitialize* from arbitrary scalar fields.

To facilitate the decoupling of advection and reinitialization, we propose a general approach for constructing a signed distance function from a set of discrete interface markers. The approach involves the construction of the signed distance function as the minimum distance from the grid to the interface. As a result, the topology of the interface is projected onto the Cartesian grid. This procedure is applied independently at every time step, without modifying the underlaying scalar field, allowing it to evolve free of reinitialization errors. Combined with high accuracy advection schemes, we demonstrate that the new approach results in substantial improvement in volume conservation and also leads to more accurate representation of interface normals and curvature.

The principle of forming the signed distance function on the basis of the minimum distance between the interface and the grid has been employed in the past [22]. Adalsteinsson and Sethian [23] consider the minimum distance from the grid to a set of interface markers for initializing the fast marching procedure. Russo and Smereka [17] used this approach to obtain a reference solution to gauge the accuracy of their reinitialization. More recently, Desjardins and Pitsch [15] defined the minimum distance with respect to piecewise linear interface segments to initialize the fast marching method [23]. A 1st order convergence of the curvature was reported by the authors by smoothing the signed distance function. The accuracy of the signed distance function and the normals was not reported.

Our approach for creating a signed distance function is based on a systematic improvement in the approximation of the minimum distance from the grid to the interface. At the first level, the interface is represented by interfaces marker that localize the interface on the grid. The second level involves a locally piecewise interface reconstruction while the third level is based on a locally smooth reconstruction. The projection of these systematically refined representations of the interface results in successively improved estimates of the signed distance function. Such projections can be applied at arbitrary distances from the interface. This approach differs from the fast marching method where the signed distance function is formed through extensional velocities on grid nodes away from the interface. In our case, each grid node inherits the topology of the projected interface directly, resulting in greater accuracy of topology representation on the grid. The projection method is shown to be robust in easily accounting for complex topological changes with an easily implementable extension to 3-D.

The article is organized as follows. In Section 2 we describe the algorithm for geometric projection. In Section 2.1 we describe the implementation for the interface advected by Lagrangian marker particles. In Section 2.2 we discuss the implementation for implicit interfaces associated with the level set function. We then demonstrate the effectiveness of the approach for two phase flow problems in Section 3.