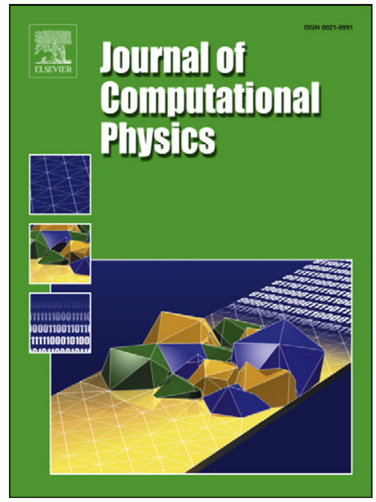
Accepted Manuscript



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PII: S0021-9991(15)00094-7
DOI: http://dx.doi.org/10.1016/j.jcp.2015.02.024
Reference: YJCPH 5728

To appear in: *Journal of Computational Physics*

Received date: 18 August 2014
Revised date: 13 February 2015
Accepted date: 15 February 2015

Please cite this article in press as: N.R. Morgan et al., A point-centered arbitrary Lagrangian Eulerian hydrodynamic approach for tetrahedral meshes, *J. Comput. Phys.* (2015), http://dx.doi.org/10.1016/j.jcp.2015.02.024

A point-centered arbitrary Lagrangian Eulerian hydrodynamic approach for tetrahedral meshes

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Abstract

We present a three dimensional (3D) arbitrary Lagrangian Eulerian (ALE) hydrodynamic scheme suitable for modeling complex compressible flows on tetrahedral meshes. The new approach stores the conserved variables (mass, momentum, and total energy) at the nodes of the mesh and solves the conservation equations on a control volume surrounding the point. This type of an approach is termed a point-centered hydrodynamic (PCH) method. The conservation equations are discretized using an edge-based finite element (FE) approach with linear basis functions. All fluxes in the new approach are calculated at the center of each tetrahedron. A multidirectional Riemann-like problem is solved at the center of the tetrahedron. The advective fluxes are calculated by solving a 1D Riemann problem on each face of the nodal control volume. A 2-stage Runge-Kutta method is used to evolve the solution forward in time, where the advective fluxes are part of the temporal integration. The mesh velocity is smoothed by solving a Laplacian equation. The details of the new ALE hydrodynamic scheme are discussed. Results from a range of numerical test problems are presented.

Keywords: Lagrangian, Eulerian, Arbitrary Lagrangian Eulerian, hydrodynamics, point-centered, Godunov, finite-element, tetrahedron

1. Introduction

Various point-centered hydrodynamic (PCH) Lagrangian approaches have been proposed [14, 25, 15, 11, 27, 52, 54, 53, 46, 68, 5]. The PCH approach is a spatially collocated method where the conservation equations for mass, momentum and total energy are solved on a control volume around the node, which is commonly termed the "dual grid". Likewise, the strain is calculated on the same dual grid. The PCH approach differs from the staggered-grid hydrodynamic approach (SGH) [62, 6, 8, 9, 70] and the cell-centered hydrodynamic (CCH) approach [1, 17, 40, 39, 7, 44, 5]. The SGH approach solves the governing equations on staggered control volumes - the cell boundary and the dual grid respectively. The CCH approach is a collocated approach that solves the governing equations on a control volume that coincides with the cell boundary. Both CCH and SGH calculate the strain on the cell boundary. The CCH approach has been successfully applied to tetrahedral meshes in [5] and triangular meshes in [4] . With the compatible Lagrangian SGH approach [6, 8], the strain calculation can be problematic on tetrahedral meshes (*i.e.* they are stiff). Scovazzi [53] performed analysis and presented numerical results demonstrating the compatible SGH approach does poorly on tetrahedral meshes. In contrast to both SGH and CCH, the PCH approach solves the governing equations on the dual grid, which are arbitrary polyhedral control volumes that have many vertices. As a result, the dual grid has far more degrees of freedom to deform than a tetrahedral cell. The PCH approach offers some advantages over alternative hydrodynamic approaches for tetrahedral meshes.

A challenge with Lagrangian methods is the mesh can tangle on problems with large deformations. It is common to use the arbitrary Lagrangian Eulerian (ALE) [29] hydrodynamic approach on problems where the mesh deforms

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