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Finite-volume solution of two-dimensional compressible flows over dynamic adaptive grids

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ABSTRACT

A novel Finite Volume (FV) technique for solving the compressible unsteady Euler equations is presented for two-dimensional adaptive grids over time dependent geometries. The interpretation of the grid modifications as continuous deformations of the underlying discrete finite volumes allows to determine the solution over the new grid by direct integration of the governing equations within the Arbitrary Lagrangian–Eulerian (ALE) framework, without any explicit interpolation step. The grid adaptation is performed using a suitable mix of grid deformation, edge-swapping, node insertion and node removal techniques in order to comply with the displacement of the boundaries of the computational domain and to preserve the quality of the grid elements. Both steady and unsteady simulations over adaptive grids are presented that demonstrate the validity of the proposed approach. The adaptive ALE scheme is used to perform high-resolution computations of the steady flow past a translating airfoil and of the unsteady flow of a pitching airfoil in both the airfoil and the laboratory reference, with airfoil displacement as large as 200 airfoil chords. Grid adaptation is found to be of paramount importance to preserve the grid quality in the considered problems.

1. Introduction

Two natural descriptions of the motion exist in continuum mechanics: the Lagrangian and the Eulerian one [1]. A third, hybrid approach is the arbitrary Lagrangian–Eulerian description of the fluid motion which combines the advantages of the other two classical approaches and possibly reduces their respective drawbacks [2,3].

In their earlier applications ALE algorithms where used to extend the capabilities of Lagrangian based solver in tackling solid mechanics problems with large deformations. Those types of algorithms usually can be schematically described by a three steps procedure. A Lagrangian/Eulerian phase during which the equations of motion are explicitly updated [4]. A rezone during which the grid quality is improved thanks to grid regularization [5] or by geometry-based node placement [6]. A remap phase, where the solution is interpolated from the old grid to the new one. This last operation is the most critical as it must be conservative, must preserve the monotonicity of the solution and should be as accurate as possible. Many approaches exist, a popular one performs an interpolation using the volumes swept by the elements during the rezone phase as weighs [7] and can be followed by a repair step to prevent under/overshoots [8]. Other techniques combine low-order inter cell fluxes with some portion of higher-order fluxes, in a flux-limiter fashion, e.g. the Flux-Corrected Remapping [9].

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More recently, a smoothing procedure was proposed to eliminate the re-mapping error in ALE schemes [10]. Different approaches exist where the solution update is coupled with the remap phase. Indeed in many applications of interest the physics equations are recast in the ALE framework implicitly accounting for the grid movement [11,12]. Remapping algorithms were successfully applied to the simulation of multi-material problems [13–15,47,48].

Recasting Eulerian schemes in the ALE framework is fairly straightforward and usually requires minor modifications to the algorithm [16], however particular care has to be taken to preserve the time accuracy [17,12,18,19]. In fact, a naïve extension of fixed-grid methods to flows in moving domains does not preserve numerical accuracy and may possibly lead to numerical instabilities [17]. Therefore, care is to be taken in both the evaluation of the local grid velocities and the definition of the geometric quantities, which cannot be chosen independently [20]. Thomas and Lombard [21] proposed to supply the discrete statement of the problem with the additional constraint of reproducing a uniform flow field exactly. This condition, known as the Geometric Conservation Law (GCL), is demonstrated to be sufficient to achieve a first-order time accuracy [22] but it is neither necessary nor sufficient for higher order accuracy [12]. Moreover, satisfying the GCL is a necessary and sufficient condition to guarantee the nonlinear stability of the integration scheme [23]. An updated review of the literature on the subject can be found in [24].

In their standard formulations, ALE methods are usually limited in their action by the occurrence of invalid elements, which poses limitations to the maximum allowable displacement. Moreover, as pointed out in [29,28,27], if re-mapping or explicit interpolation over different grids is applied to account for the grid topology alteration in time, difficulties arise in including multi-step time integration algorithm. For very large displacement of the boundaries, it is possible to locally change the topology of the grid without modifying the number of nodes [25–27], although preserving the grid quality and the desired spacing is not straightforward.

In a previous paper the authors presented a node-centered finite-volume ALE solver for grids undergoing edge-swapping [27], where the modifications occurring in the shape of the finite volumes due to the changes in the topology are recast in a continuous fashion. Such approach makes it possible to compute the solution at the subsequent time level simply by integrating the governing equations and avoiding the need of an explicit remap phase. In the present work the same finite-volume solver is extended to the case of grid refinement and coarsening. Similarly to the case of edge swapping, the insertion or deletion of a node is described in terms of continuous deformation of the finite volumes associated to the computational grid. Therefore, when a vertex is inserted a new finite volume appears while it disappears when a vertex is removed.

The key idea is to give an interpretation of the changes in the topology that occur in the time lapse from t^n and t^{n+1} as continuous deformation of the finite volumes performed within the same time interval. The area swept by the interfaces is split into two separate contributions: the deformation one, arising from the continuous (in time) grid movement and distortion and the adaptation one, in which additional numerical fluxes are included to account for the changes in the topology. The proposed approach avoids the introduction of any explicit interpolation step between grids with different topologies and instead makes use of the ALE approach, as it is commonly done when only grid deformation is used. Admittedly, the application of ALE mapping is equivalent to an interpolation step; however, its application does not require any special treatment to ensure appropriate accuracy, conservativeness and preservation of function signs. Since cross-grid interpolation is avoided [28,29], the implementation of multi-step high-order schemes for time integration, e.g. BDF schemes, is straightforward, as discussed in [27].

Future extensions to viscous, thermal conducting fluid are expected to be straightforward, since the ALE formulation does not modify the viscous and thermal conductivity contributions [2,3]. Care must be taken however in the proximity of the body surface, where very stretched boundary-layer grids made of non-simplectic elements are commonly used. Note that ALE schemes implementing shock-capturing techniques, including the present one, can be easily extended to deal with multi-material interface by e.g. the shock-capturing method of Abgrall [30] and modifications of it [31,32]. As an alternative, thanks to the large-displacement capability of the present scheme, the material interfaces can be explicitly advected within the ALE formulation and boundary-conforming grids can be used to represent it [33,34]. Extension to multi-material flows will be the focus of future research activities.

The present paper is structured as follows. The grid update strategy is briefly described in Section 2. In Section 3 the edge-based ALE solver is described for the case of a non-adaptive grid. A brief description of the time integration procedures is also given in Section 3.2. In Section 4 are presented the modifications to the scheme required to account for the occurrence of edge-swapping, node insertion or deletion. In Section 4.5, implementation details are reported. In Section 5, numerical experiments are reported. In Sections 5.1 and 5.2 the proposed scheme is applied to the pseudo-steady case of a translating airfoil and the unsteady case of a translating and oscillating airfoil, respectively. Computations are carried out in both the airfoil and the laboratory reference frame to demonstrate the suitability of the present approach.

2. Grid update strategy

In the present work, grid adaptation techniques are used to preserve a high quality of the mesh and to enforce the desired element spacing distribution in numerical simulations of compressible flows involving large displacements of the boundaries. To this purpose, a suitable mix of techniques is adopted to displace the nodes and to locally modify the topology