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An asymptotic preserving unified gas kinetic scheme for gray radiative transfer equations

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ABSTRACT

The solutions of radiative transport equations can cover both optical thin and optical thick regimes due to the large variation of photon's mean-free path and its interaction with the material. In the small mean free path limit, the nonlinear time-dependent radiative transfer equations can converge to an equilibrium diffusion equation due to the intensive interaction between radiation and material. In the optical thin limit, the photon free transport mechanism will emerge. In this paper, we are going to develop an accurate and robust asymptotic preserving unified gas kinetic scheme (AP-UGKS) for the gray radiative transfer equations, where the radiation transport equation is coupled with the material thermal energy equation. The current work is based on the UGKS framework for the rarefied gas dynamics [14], and is an extension of a recent work [12] from a one-dimensional linear radiation transport equation to a nonlinear two-dimensional gray radiative system. The newly developed scheme has the asymptotic preserving (AP) property in the optically thick regime in the capturing of diffusive solution without using a cell size being smaller than the photon's mean free path and time step being less than the photon collision time. Besides the diffusion limit, the scheme can capture the exact solution in the optical thin regime as well. The current scheme is a finite volume method. Due to the direct modeling for the time evolution solution of the interface radiative intensity, a smooth transition of the transport physics from optical thin to optical thick can be accurately recovered. Many numerical examples are included to validate the current approach.

1. Introduction

This paper is about the development of a numerical scheme for the solution of the time-dependent gray radiative transfer equations. It is well-known that the gray radiative transfer equations are modeling in the kinetic scale, where the dynamics of photon transport and collision with material is taken into account. The wide applications of this system include astrophysics, inertial confinement fusion, high temperature flow systems, and many others. Due to the importance and complexity of the system, its study attracts much attention from national laboratories and academic institutes.

The gray radiative transfer equations model the radiation energy transport and the energy exchange with the background material. The properties of the background material influence greatly on the behavior of radiation transfer. For a low opac-

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ity (background) material, the interaction between the radiation and material is weak, and the radiation propagates in a transparent way. The numerical method in this regime for the streaming transport equation is well defined by tracking the rays. However, for a high opacity (background) material, there is severe interaction between radiation and material with a diminishing photon mean free path. As a result, the diffusive radiative behavior will emerge. In order to solve the kinetic scale based radiative transfer equations numerically, the spatial mesh size should be comparable to the photon's mean-free path, which is very small in the optical thick regime. Consequently, the simulation in this regime is associated with huge computational cost. To remedy these difficulties, one of the approaches is to develop the so-called asymptotic preserving (AP) scheme for the kinetic equation. When holding fixed mesh size and time step and letting the Knudsen number go to zero, the AP scheme should automatically recover the diffusion solution. The aim of this paper is to develop such an AP scheme for the gray radiation transfer equations. At the same time, the solution in the optical thin regime can be captured as well.

AP schemes were first studied in the numerical solution of steady neutron transport problems by Larsen, Morel and Miller [9], Larsen and Morel [8], and then by Jin and Levermore [4,5]. For unsteady problems, the AP schemes were constructed based on a decomposition of the distribution function between an equilibrium part and its non-equilibrium derivation, see Klar [7], and Jin, Pareschi and Toscani [6] for details. Based on the unified gas kinetic scheme (UGKS) framework [14], a rather different approach has recently been proposed by Mieussens for a linear radiation transport model [12]. The UGKS is a multi-scale direct modeling method with coupled particle transport and collision. An integral solution of the kinetic model equation has been used to construct the flow evolution around a cell interface for the flux evaluation in the finite volume scheme. This time evolution solution covers the flow physics from the kinetic scale particle free transport to the hydrodynamic scale wave propagation or diffusion limit, and the real solution used in a specific regime depends on the ratio of time step to the local particle collision time. As a result, both kinetic and hydrodynamic limiting solutions can be obtained accurately, as well as the solutions in the transition regime. Due to the un-splitting treatment for the transport and collision in UGKS, the cell size and time step used in the scheme are not limited by the particle mean free path and collision time [3,1].

In this paper, for the first time an asymptotic preserving unified gas kinetic scheme (AP-UGKS) will be developed for the gray radiation transfer equations, which are composed of radiation transport and material energy equation. The discrete-ordinate method is used to discretize the angle direction of the photon's movement. The integral solution of the radiation intensity at a cell interface is constructed for the flux evaluation. With the angular integration on the radiation transport equation and the direct use of the material energy equation, for the gray radiation transfer system, two nonlinearly coupled macroscopic variables-based equations are solved at the next time level for the solutions of the radiation energy and material temperature inside each cell. For this coupled system, the resulting discrete equations give a standard five points diffusion scheme for the material temperature in a two dimensional case with Cartesian coordinates. With the updated radiation energy inside each cell at the next time level, the radiation intensity is explicitly obtained through the interface flux and the inner cell source term treatment. Many numerical test cases are included to validate the current approach.

This paper is organized as follows. Section 2 gives the model equations of gray radiation transfer. Section 3 reviews the construction of the unified scheme for the linear radiative equation. Section 4 presents AP-UGKS for the gray radiative transfer equations. In Section 5, many numerical tests are included to demonstrate the accuracy and robustness of the new scheme. The last section is the conclusion.

2. System of the gray radiative transfer equations

The gray radiative transfer equations describe the radiative transfer and the energy exchange between radiation and material. The equations can be written in following scaled form:

$$\begin{cases} \dfrac{\epsilon^2}{c}\dfrac{\partial I}{\partial t} + \epsilon\vec{\Omega}\cdot\nabla I = \sigma\left(\dfrac{1}{4\pi}acT^4 - I\right), \\ \epsilon^2 C_v \dfrac{\partial T}{\partial t} \equiv \epsilon^2\dfrac{\partial U}{\partial t} = \sigma\left(\displaystyle\int I d\vec{\Omega} - acT^4\right). \end{cases} \tag{2.1}$$

Here the spatial variable is denoted by $\vec{r}$, $\vec{\Omega}$ is the angular variable, and t is the time variable, $I(\vec{r}, \vec{\Omega}, t)$ is the radiation intensity, $T(\vec{r}, t)$ is the material temperature, $\sigma(\vec{r}, T)$ is the opacity, a is the radiation constant, and c is the speed of light, $\epsilon > 0$ is the Knudsen number, and $U(\vec{r}, t)$ is the material energy density. For the simplicity of presentation, we have omitted the internal source and scattering terms in (2.1).

Eq. (2.1) is a relaxation model for the radiation intensity to the local thermodynamic equilibrium, in which the emission source is a Planckian at the local material temperature:

$$\frac{1}{4\pi}\sigma acT^4.$$

The material temperature $T(\vec{r}, t)$ and the material energy density $U(\vec{r}, t)$ are related by