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A hyperbolic-equation system approach for magnetized electron fluids in quasi-neutral plasmas

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ABSTRACT

A new approach using a hyperbolic-equation system (HES) is proposed to solve for the electron fluids in quasi-neutral plasmas. The HES approach avoids treatments of cross-diffusion terms which cause numerical instabilities in conventional approaches using an elliptic equation (EE). A test calculation reveals that the HES approach can robustly solve problems of strong magnetic confinement by using an upwind method. The computation time of the HES approach is compared with that of the EE approach in terms of the size of the problem and the strength of magnetic confinement. The results indicate that the HES approach can be used to solve problems in a simple structured mesh without increasing computational time compared to the EE approach and that it features fast convergence in conditions of strong magnetic confinement.

1. Introduction

Electron fluids in quasi-neutral plasmas appear in many practical plasma simulations such as space propulsion, astrophysics, and plasma processing [1–4]. In simulations of quasi-neutral plasmas, one commonly uses the plasma approximation, in which the space potential is solved using the electron conservation equations, rather than by using Gauss's law [5]. In conventional methods, the equations of conservation of mass and conservation of momentum are integrated into one elliptic equation (EE), and this equation is solved for the space potential as a boundary value problem. However, this EE is an "anisotropic diffusion equation" and, owing to magnetic confinement, solving this equation is complicated by the following factors: 1) the anisotropy resulting from the large difference in diffusion coefficient in each direction and 2) the instability caused by cross-diffusion terms. The cross-diffusion terms are especially very difficult to handle because they cause failure of the diagonal dominance of the coefficient matrix.

One approach to avoid the cross-diffusion terms is to use a magnetic-field-aligned mesh (MFAM) [6]. Because the cross-diffusion terms stem from the angle between the magnetic lines of force and the computational mesh, they can be neglected if the mesh is precisely aligned with the magnetic lines of force. However, using an MFAM makes it impossible to use a structured mesh for the body-fitted coordinate system and complicates the evaluation of fluxes on the boundaries. Furthermore, once the magnetic field induced by the plasma flow is solved, one needs to reconstruct the mesh with changing magnetic lines of force. Thus, a practical application of an MFAM is associated with cumbersome steps in coding and implementation.

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Recently, a new approach to solving diffusion equations that utilizes a hyperbolic-equation system (HES) has been proposed [7,8]. This approach divides the diffusion equation into an HES that contains only advection terms by introducing some pseudo-time advancement terms. The mathematical strategy of this approach entails dividing a second or higher order differential equation into a set of first-order differential equations by introducing new variables, including a first derivative of another variable. This approach is supposed to be effective in solving the anisotropic diffusion equation of electron fluids including cross-diffusion terms.

The purpose of this paper is to find a robust and efficient method using an HES for electron fluids in quasi-neutral plasmas. The fundamental equations of electron fluids in quasi-neutral plasmas are introduced and the HES construction for the equations of electron fluids is discussed. Numerical computational fluid dynamics techniques are applied to the HES approach to reduce the computation cost. The HES approach is validated through a two-dimensional numerical test. Also, the performance of the HES approach is analyzed by comparing the computation time of the HES approach with that of the approach using an EE with an MFAM.

2. Modeling of electron fluids in quasi-neutral plasmas

2.1. Fundamental equations

The fundamental equations treated herein are the two-dimensional conservation equations of electron mass and electron momentum in a quasi-neutral plasma. To solve for the space potential, one generally assumes the plasma approximation, in which the space potential is calculated through conservation equations of electrons, rather than through Gauss's law [5]. The conservation of mass is formulated as follows:

$$\nabla \cdot (m_e n_e \vec{u}_e) = m_e n_e \nu_{ion}, \tag{1}$$

where m_e, n_e, $\vec{u}_e$, and ν_{ion} are the electron mass, electron number density, electron velocity, and ionization collision frequency, respectively. Under the quasi-neutrality assumption, the electron number density is equal to the ion number density. The ion number density is derived through the computation of the ion flow, which is not treated in this paper. Thus, the electron number density is treated as a given distribution. The time-derivative term of the electron number density is not included in Eq. (1) because electrons are sufficiently mobile to instantaneously achieve quasi-neutrality. Therefore, the electron number density is regarded as a time-invariant quantity. Because the electron density is given as a time-invariant quantity, the electron fluids in quasi-neutral plasmas have characteristics similar to those of incompressible fluids.

Conservation of momentum is derived from the Navier–Stokes equation,

$$m_e n_e \frac{\partial \vec{u}_e}{\partial t} + m_e n_e (\vec{u}_e \cdot \nabla)\vec{u}_e = -\nabla(e n_e T_e) + e n_e \nabla\phi - e n_e \vec{u}_e \times \vec{B} - m_e n_e \nu_{col} \vec{u}_e, \tag{2}$$

where e, T_e, ϕ, $\vec{B}$, and ν_{col} are the elemental charge, electron temperature, space potential, magnetic flux density, and electron-neutral total collision frequency, respectively. The forces working on the fluid element are pressure, the electrostatic force, the electromagnetic force, and the collisional force from the collisions between electrons and neutral particles. In this paper, electron temperature is supposed to be a given spatial distribution and to be a time-invariant quantity. This assumption is often used in the simulations of space plasma [9]. If one needs to treat the electron temperature as a time-dependent variable, the electron temperature is calculated by including the energy conservation equation into the system [10]. In this paper, the energy conservation equation is excluded from the system to focus on a simple anisotropic diffusion equation. Here, the effects of electron–electron collision and electron–ion collision are neglected because generally their collision frequencies are much lower than the electron-neutral collision frequency. Furthermore, because of the large number of collisions, the inertia of the electron fluid is negligibly small, and the force working on the fluid element is balanced, so

$$0 = -\nabla(e n_e T_e) + e n_e \nabla\phi - e n_e \vec{u}_e \times \vec{B} - m_e n_e \nu_{col} \vec{u}_e. \tag{3}$$

After linear conversion of Eq. (3), the electron flux in tangential ($\parallel$) and orthogonal ($\perp$) directions of the magnetic lines of force can be described by using the electron mobility μ:

$$n_e \begin{pmatrix} u_\parallel \\ u_\perp \end{pmatrix} = n_e [\mu]_{mag} \begin{pmatrix} \nabla_\parallel \phi \\ \nabla_\perp \phi \end{pmatrix} - [\mu]_{mag} \begin{pmatrix} \nabla_\parallel (n_e T_e) \\ \nabla_\perp (n_e T_e) \end{pmatrix}, \tag{4}$$

$$[\mu]_{mag} = \begin{bmatrix} \mu_\parallel & \\ & \mu_\perp \end{bmatrix} = \begin{bmatrix} \frac{e}{m_e \nu_{col}} & \\ & \frac{\mu_\parallel}{1+(\mu_\parallel |B|)^2} \end{bmatrix}. \tag{5}$$

Eq. (4) is Ohm's law for electron current on a coordinate system fitted to magnetic lines of force. In the derivation of Eq. (4), the $E \times B$ drift and the diamagnetic drift are neglected by assuming symmetry in one orthogonal direction with the magnetic lines of force. $\mu_\perp$ in Eq. (5) is based on the classical diffusion model, and this model can be modified for better reflection of magnetic confinements such as the Bohm diffusion model, depending on the situation [11]. The electron mobility on a computational mesh is derived by rotating $[\mu]_{mag}$ with the angle between the magnetic lines of force and the computational mesh. Thus, we have the following equations.