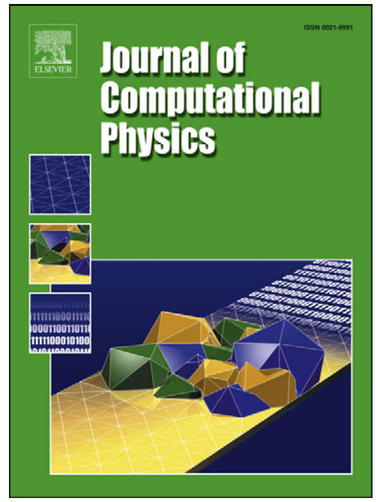
Accepted Manuscript



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PII: S0021-9991(14)00760-8
DOI: 10.1016/j.jcp.2014.11.008
Reference: YJCPH 5572

To appear in: *Journal of Computational Physics*

Received date: 29 December 2013
Revised date: 22 October 2014
Accepted date: 6 November 2014

A new spectral difference method using hierarchical polynomial bases for hyperbolic conservation laws

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Abstract

To solve hyperbolic conservation laws, a new method is developed based on the spectral difference (SD) algorithm. The new scheme adopts hierarchical polynomials to represent the solution in each cell instead of Lagrange interpolation polynomials used by the original one. The degree of freedoms (DOFs) of the present scheme are the coefficients of these polynomials, which do not represent the states at the solution points like the original method. Therefore, the solution points defined in the original SD scheme are discarded, while the flux points are preserved to construct a Lagrange interpolation polynomial to approximate flux function in each cell. To update the DOFs, differential operators are applied to the governing equation as well as the Lagrange interpolation polynomial of flux function to evaluate first and higher order derivatives of both solution and flux at the centroid of the cell. The stability property of the current scheme is proved to be the same as the original SD method when the same solution space is adopted. One dimensional methods are always stable by the use of zeros of Legendre polynomials as inner flux points. For two dimensional problems, the introduction of Raviart-Thomas spaces for the interpolation of flux function proves stable schemes for triangles. Accuracy studies are performed with one - and two-dimensional problems. P-multigrid algorithm is implemented with orthogonal hierarchical bases. The results verify the high efficiency and low memory requirements of implementation of p-multigrid algorithm with the proposed scheme.

Keywords: spectral difference, Raviart-Thomas space, hierarchical polynomials, Taylor expansion, orthogonal bases, p-multigrid, hyperbolic conservation law

1. Introduction

In the recent years, some problems of great interest, such as computational aeroacoustics (CAA), direct numerical simulations (DNS) and large eddy simulation (LES) of turbulence, require numerical methods with low dispersion and dissipation. In response to these requests, some high-order methods have been proposed and developed, including essentially nonoscillatory (ENO) and weighted essentially nonoscillatory (WENO) schemes [1–6], discontinuous Galerkin (DG) method [7–11], spectral volume (SV) method [16–21], spectral difference (SD) method [24–27]. Those work have been summarized comprehensively by Ekaterinaris [29] and Wang [30].

As the pioneer of high-order numerical methods for solution of partial difference equations, the DG method has become popular in CFD community in recent years because of some attractive features, such as local conservative, geometric flexibility, compactness which makes it ideally suited for parallel computing. The DG algorithm was first introduced by Reed and Hill in 1973 to solve neutron transport equation [7]. A major development of the DG scheme to solve hyperbolic conservation laws was carried out by Cockburn, Shu and their collaborators in a series of papers [8–11] on the Runge-Kutta DG (RKDG) method, which

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