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Multi-frequency subspace migration for imaging of perfectly conducting, arc-like cracks in full- and limited-view inverse scattering problems

Won-Kwang Park

Department of Mathematics, Kookmin University, Seoul 136-702, Korea

A R T I C L E I N F O

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A B S T R A C T

Multi-frequency subspace migration imaging techniques are usually adopted for the non-iterative imaging of unknown electromagnetic targets, such as cracks in concrete walls or bridges and anti-personnel mines in the ground, in the inverse scattering problems. It is confirmed that this technique is very fast, effective, robust, and can not only be applied to full- but also to limited-view inverse problems if a suitable number of incidents and corresponding scattered fields are applied and collected. However, in many works, the application of such techniques is heuristic. With the motivation of such heuristic application, this study analyzes the structure of the imaging functional employed in the subspace migration imaging technique in two-dimensional full- and limited-view inverse scattering problems when the unknown targets are arbitrary-shaped, arc-like perfectly conducting cracks located in the two-dimensional homogeneous space. In contrast to the statistical approach based on statistical hypothesis testing, our approach is based on the fact that the subspace migration imaging functional can be expressed by a linear combination of the Bessel functions of integer order of the first kind. This is based on the structure of the Multi-Static Response (MSR) matrix collected in the far-field at nonzero frequency in either Transverse Magnetic (TM) mode (Dirichlet boundary condition) or Transverse Electric (TE) mode (Neumann boundary condition). The investigation of the expression of imaging functionals gives us certain properties of subspace migration and explains why multi-frequency enhances imaging resolution. In particular, we carefully analyze the subspace migration and confirm some properties of imaging when a small number of incident fields are applied. Consequently, we introduce a weighted multi-frequency imaging functional and confirm that it is an improved version of subspace migration in TM mode. Various results of numerical simulations performed on the far-field data affected by large amounts of random noise are similar to the analytical results derived in this study, and they provide a direction for future studies.

1. Introduction

The main purpose of inverse scattering problems such as non-destructive evaluation is identifying unknown characteristics of defects such as size, location, shape, and electric and magnetic properties. Among them, the identification of the shape of arbitrary-shaped cracks in structures such as bridges, concrete walls, and machines is an interesting problem that

E-mail address: parkwk@kookmin.ac.kr.

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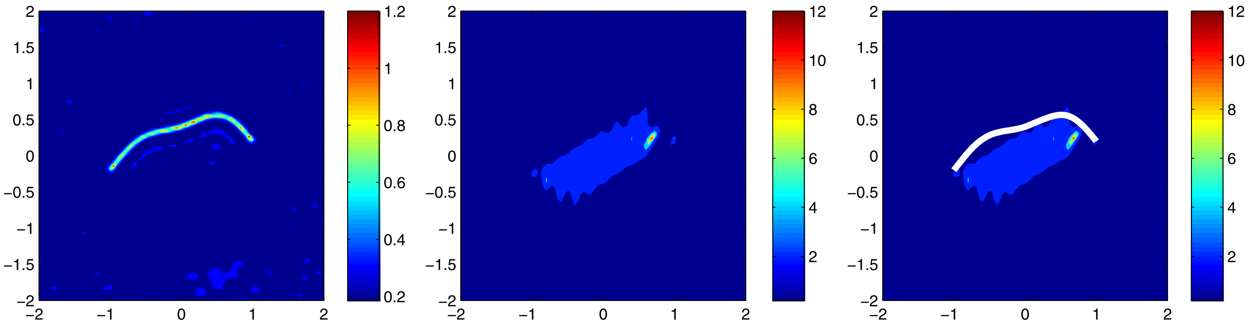

(a) full-view case (b) limited-view case (c) true shape

Fig. 1. Reconstruction of arbitrary-shaped extended perfectly conducting crack via MUSIC-type algorithm [48]. In the full-view problem, MUSIC offers a very good result, but a very poor result appears in the limited-view problem.

can be encountered in everyday life. Unfortunately, because of the intrinsic difficulties of its ill-posedness and nonlinearity, this problem cannot be successfully resolved.

Presently, many remarkable inversion techniques and corresponding computational environments are being developed and established to solve this problem. The main approach for solving this problem is based on the Newton-type iteration method, i.e., obtaining the shape of the target (minimizer), which minimizes the discrete norm (generally, L^2-norm) between the measured scattered or far-fields in the presence of true and man-made targets. Iteration-based techniques such as the level-set method and optimization algorithm have been successfully applied to identify the number, location, shape, and topological properties of cracks with a small number of directions of the incident and scattered field data as shown in many works [2,19,23,32,51,57]. Nevertheless, obtaining a good initial guess close to the target, estimating *a priori* information such as length, location, material properties, selecting appropriate regularization terms that are significantly dependent on the problem, and evaluating the so-called Fréchet (or domain) derivative must be considered beforehand as described in [34]. If one of these conditions is not fulfilled, one may encounter various problems such as the phenomenon of non-convergence, the occurrence of local minimizer problems, and the requirement of large computational costs because of the large number of iteration procedures. To overcome these problems, a simultaneous reconstruction algorithm has been developed [26,54]. However, this type of algorithm still has limitations; for example, it must be performed with a good initial guess and it is generally very slow. Hence, this fact indicates that the development of an alternative fast algorithm and a corresponding rigorous mathematical theory for obtaining a good initial guess in the beginning stages of the iteration procedure is necessary.

Correspondingly, for an alternative, various non-iterative imaging algorithms have been developed and successfully applied to various inverse problems. Based on the calculations of the inverse Fourier transform, a variational algorithm was proposed in [12,13,15]. Based on these studies, this algorithm produces very good results; however, it is still restricted to identifying locations of small inclusions, and hence its extension to the reconstruction of arbitrary-shaped electromagnetic targets is a topic for further research. In [18,21,31], a linear sampling method was developed and applied for determining the locations and shapes of unknown scatterers; however, this approach requires a large number of directions of the incident and scattered fields, and it is not considered for the limited-view inverse scattering problems. MUltiple SIgnal Classification (MUSIC), which is closely related to the linear sampling method (see [20]), is also applied to various inverse scattering problems for full- and limited-view problems for imaging arbitrary-shaped thin, penetrable electromagnetic scatterers, cracks, and extended targets in two- and three-dimensional spaces [10,11,8,14,28,30,41,45,48–50]. Based on the results in [3,48,50], the MUSIC algorithm still requires a significant number of directions of the incident and scattered field data to obtain an acceptable result, and it does not guarantee the complete shape of targets because of the intrinsic resolution limit related to the half applied wavelength. In recent works [10,45,49], the MUSIC algorithm was applied to limited-view inverse problems; however, it yields incorrect locations of small inhomogeneities and the shape of extended targets (for example, see Fig. 1) throughout the structure of the MUSIC imaging function derived in [30]; the reason for this phenomenon has been mathematically proved. Originally, a topological derivative strategy was applied in the shape optimization problems; however, in recent works, it has been confirmed that this strategy is a non-iterative imaging technique and can be successfully applied to various inverse scattering problems (see [6,16,19,36,40,43,46,47,56] and the references therein). The remarkable advantages of topological derivative-based imaging techniques are that they produce good results even with a small number of directions of incident field data, and they are robust against the significant amount of random noise. However, throughout the derivation of topological derivatives in [14,43,46,47], this strategy covers only the case of full-view inverse scattering problems, and a small number of directions of incident fields must span the unit circle $\mathbb{S}^1$, which means that it is especially vulnerable in the case of limited-view inverse problems.

Given that the number of directions of the incident field and corresponding scattered fields are sufficiently large, Kirchhoff and subspace migration imaging techniques operated at single and multiple time-harmonic frequency have been successfully applied not only for the full- but also for the limited-view inverse scattering problem. Related works can be found in [8,17,27,29,35,42,44,45,49,52] and the references therein. These techniques are very robust to random noise