



A comparative study of diffraction of shallow-water waves by high-level IGN and GN equations



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ABSTRACT

This work is on the nonlinear diffraction analysis of shallow-water waves, impinging on submerged obstacles, by two related theories, namely the classical Green–Naghdi (GN) equations and the Irrational Green–Naghdi (IGN) equations, both sets of equations being at high levels and derived for incompressible and inviscid flows. Recently, the high-level Green–Naghdi equations have been applied to some wave transformation problems. The high-level IGN equations have also been used in the last decade to study certain wave propagation problems. However, past works on these theories used different numerical methods to solve these nonlinear and unsteady sets of differential equations and at different levels. Moreover, different physical problems have been solved in the past. Therefore, it has not been possible to understand the differences produced by these two sets of theories and their range of applicability so far. We are thus motivated to make a direct comparison of the results produced by these theories by use of the same numerical method to solve physically the same wave diffraction problems. We focus on comparing these two theories by using similar codes; only the equations used are different but other parts of the codes, such as the wave-maker, damping zone, discretion method, matrix solver, etc., are exactly the same. This way, we eliminate many potential sources of differences that could be produced by the solution of different equations. The physical problems include the presence of various submerged obstacles that can be used for example as breakwaters or to represent the continental shelf. A numerical wave tank is created by placing a wavemaker on one end and a wave absorbing beach on the other. The nonlinear and unsteady sets of differential equations are solved by the finite-difference method. The results are compared with different equations as well as with the available experimental data.

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1. Introduction

The Green–Naghdi (hereafter, GN) theory [14,15] has been used to analyze nonlinear free-surface flows, see e.g., [11]. The GN theory can also be looked at as it adopts a shape function representation that approximates the vertical structure of the velocity field [5]. The governing equations are the depth-integrated form of Euler's equations. Both the bottom boundary condition and the nonlinear free-surface boundary conditions are satisfied exactly. Because no other assumptions and

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approximations are introduced, the GN equations are universally valid and can be applied to any water depth, to regular and irregular waves, and incorporate effects of varying bottom topography and vorticity (rotational flow).

The irrotational Green–Naghdi (IGN) equations derived by Kim et al. [16], on the other hand, have been applied to simulate ocean waves of extreme wave heights [18] in deep waters. The IGN equations have also been derived for finite water depth conditions and have been numerically tested to show their self-convergence and accuracy [17,20]. The original (or classical) GN governing equations are not limited to irrotational flows. The GN model was extended to deep-water waves by Webster and Kim [28] and Xu et al. [30] in two and three dimensions, respectively. See also Ertekin and Sundararaghavan [10].

The different degrees of complexity in the GN theories are distinguished by ‘levels’. For shallow water waves, Level I GN theory (here after, GN-1) was used to simulate some nonlinear wave flows [9,29]. Nadiga et al. [23] studied the overturns of isopycnal surfaces by use of the GN-1 model. Their results show that the GN approximation, which makes no assumptions on the size of the nonlinearity, is verified to be better than the generalized Boussinesq (gB) approximation used in cases where the bottom topography is “large” or when the bottom topography is “moderate” but the flow is transcritical. Level II GN theory (here after, GN-2) has been used to simulate shallow water and nonlinear wave propagation problems in two dimensions [5,6]. Due to the rapid increase in algebraic complexity at higher levels, the GN models up to Level III have been derived, but only a limited number of applications exist [26,30]. This is even more so in the application of the GN theory to 3-dimensional problems except by the use of the GN-1 equations in a couple of cases (see e.g., Neil and Ertekin [24], Ertekin and Sundararaghavan [10]). The well-known MCC model [4] used in internal wave simulations is the GN-1-1 model which means both the upper layer and the lower layer are using the GN-1 model separately.

Different levels of the classical GN equations have different dispersive properties (see e.g., Webster et al. [27] for water waves, and Kim and Ertekin [19] for hydroelastic waves); this is also true for different levels of the IGN equations (see e.g., Kim and Ertekin [18], Kim et al. [17]). In the past, it was very difficult to apply the classical GN equations at levels higher than II. Webster et al. [27] simplified the structure of the GN equations, thus enabling the utility of the high-level GN equations (higher than Level II). The high-level GN equations have been shown to be more accurate in recent numerical calculations [32,33].

Le Métayer et al. [21] proposed a hybrid numerical method using a Godunov-type scheme to solve the Green–Naghdi model describing dispersive shallow-water waves. The model they utilized is the GN-1 model which is able to simulate the weakly dispersive shallow-water waves. Chazel et al. [3] derived a three-parameter Green–Naghdi model optimized for uneven bottoms. Actually, their derivation is based on the GN-1 model. [33] tested the same case simulated in Chazel et al. [3], and the results show that the converged GN results (GN-5) are much better compared with the model given in Chazel et al. [3]. Bonneton et al. [1] proposed a hybrid finite-volume and finite-difference splitting approach for the fully nonlinear and weakly dispersive Green–Naghdi model. The results show that their approach gives a good account of all the processes of wave transformation in coastal areas: shoaling, wave breaking and run-up. We should mention that the GN equations they used again do belong to the GN-1 theory. When the level goes up, the GN model becomes a strongly dispersive model as was shown by Shields and Webster [26].

Zhang et al. [31] showed that properties of Boussinesq–Green–Naghdi equations may be substantially improved for a given order of approximation using asymptotic rearrangements. This improvement is accomplished by using a large number of degrees of freedom inherent in the definitions of the polynomial basis functions either to match additional terms in a Taylor series, or to minimize errors over a range. Their last test case (in Section 6 shown in their paper) is the same case which simulated by Zhao et al. [33] (in Section 5.1). But, we find that the results from Zhang et al. [31] are not as good as the high level GN results (such as GN-5) which were presented in Zhao et al. [33]. The high-level GN models are strongly dispersive and strongly nonlinear and therefore there is a definite need to use them in more applications, and they can clearly be used to simulate the diffraction of water waves in finite depth. Whereas, the high-level GN model employed here cannot simulate wave breaking and wave overturning because the conservation equations are satisfied (exactly) in the depth-integrated sense.

The main motivation for this research is, therefore, to compare the high-level (converged) GN results and IGN results by use of the same numerical scheme and the codes except the equations themselves to have a fair comparison to understand their applicability. This is necessary as the GN or IGN theories are not based on any perturbation expansion and therefore their applicability can only be judged by comparison with other theories and experimental data.

In Section 2, the high-level GN equations are introduced. Section 3 provides the high-level IGN equations. Section 4 introduces the algorithm used here. Boundary conditions are discussed in Section 5. The numerical test cases use the high-level GN and IGN equations and they are presented in Section 6. These are followed by the conclusions we reach in Section 7.

2. GN theory

The simplified governing equations for the motion of a thin sheet of fluid are provided by the GN theory [27,33]. There are basically four sets of equations of the GN theory at each level; these are the ones obtained by Green and Naghdi [15], Shields and Webster [26], Demirbilek and Webster [6], and by Webster et al. [27]. They appear different but they are essentially the same as they all follow the main character of the GN theory; that the equations provide solutions that

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