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One-dimensional characteristic boundary conditions using nonlinear invariants

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ABSTRACT

A new treatment of boundary conditions using the flow decomposition in characteristics is derived for inviscid one-dimensional flows using nonlinear invariants. The new set of equations is equivalent to the relations of Thompson (1987) [5] but it has the advantage to provide a physical interpretation of the characteristics for nonlinear perturbations. This interpretation is a major advantage to deal with the nonlinear injection of waves in the domain. In particular, the limitation of the standard relations to the injection of waves without nonlinear interactions on the boundary is addressed. Associated errors are evaluated analytically and numerically and a clear improvement of the results is demonstrated with the new expressions. To avoid a drift of the mean values, relaxation terms are usually added in the relations and the boundary conditions become almost non-reflecting. The consequences of these relaxation terms on outgoing and ingoing waves are widely investigated in the present paper and a nonlinear correction is proposed to recover perfectly non-reflecting conditions without drift. To end, simulations are performed on the generation of indirect combustion noise through a nozzle to illustrate the advantages of the new formulation.

1. Introduction

In the frame of numerical simulation for fluid dynamics, the truncation of the numerical domain at open boundaries with reliable boundary conditions is a key issue to provide high-quality simulations representative of experimental configurations. Such boundary conditions are expected to avoid the contamination of the physical computed flow by spurious numerical reflections and to allow the injection of flow perturbations inside the numerical domain. Considerable efforts have been performed during the last decades to provide numerical treatments of the boundaries that ensure transparent boundary conditions and possible flow forcing. Such transparent conditions are especially important nowadays with the use of high-order numerical schemes, where numerical damping is very low and waves need to leave the domain without numerical reflections at the risk of trapping spurious oscillations inside the computed region and leading to nonphysical solutions.

Non-reflecting boundary conditions are developed for linear perturbations for instance by Bayliss and Turkell [1] and Engquist and Majda [2,3]. Hedstrom [4] is the first to propose a nonlinear condition for non-reflection using one-dimensional Euler equations. This approach, based on the evaluation of the ingoing and outgoing characteristic waves at the boundaries, is later reformulated and completed by Thompson for two- and three-dimensional flows [5,6]. All these works deal

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with inviscid equations and are therefore unsuitable for direct numerical simulations or large-eddy simulations of high-Reynolds number, turbulent viscous flows. To perform such simulations, Poinsot and Lele [7] extend the method developed by Thompson [5] to Navier–Stokes equations. The formulation of Poinsot and Lele, referred to as Navier–Stokes Characteristics Boundary Conditions (NSCBC), consists in recasting the Navier–Stokes equations to separate the contributions of the one-dimensional inviscid characteristic waves and additional conditions referring to transverse terms and viscous and diffusion effects. The interested reader may find a detailed methodology for the implementation of the NSCBC approach in Poinsot and Veynante [8] and a more extensive review about the modeling of non-reflective boundary conditions in Colonius [9].

An extended discussion on the implementation of time-dependant flow variables on the boundaries using the characteristics method is first addressed by Poinsot and Lele [7]. To determine the amplitudes of the incoming waves, they assume the inviscid flow to be locally oriented in the direction normal to the boundary and write a set of local one-dimensional inviscid (LODI) equations that relate the time derivative of the flow variables with the characteristics. They also provide a physical interpretation of the characteristics for perturbations in linear regime. The implementation of Poinsot and Lele has proved to be very efficient for a large number of practical configurations. Limitations may however be observed especially when transverse terms are large on the boundaries. To deal with that problem, Yoo et al. [10] and Lodato et al. [11] extend the NSCBC method to two-dimensional and three-dimensional flows, respectively, following the initial approach of Thompson [5,6]. In particular, Lodato et al. [11] give a detailed discussion on the treatment of edges and corners, where a coupling exists between the normal and transverse characteristic waves from the different boundaries.

A last issue for the implementation of transparent boundary conditions with the NSCBC method is the imposition of the mean flow values. Physically, the information corresponding to the mean flow quantities is provided by the ingoing characteristic waves reflecting from regions far away from the domain considered in the simulation [7]. Perfectly non-reflecting boundary conditions correspond to setting all ingoing waves to zero and in that case the mean flow values are not imposed anymore. The problem might be therefore ill-posed and a drift of the mean flow may occur. This problem has been addressed by several authors such as Rudy and Strikwerda [12,13], Hagstrom and Hariharan [14] and Keller and Givoli [15] and a proposed solution is to use corrective terms through linear relaxation coefficients to provide almost non-reflecting boundary conditions.

To end, different techniques can be used for the injection of perturbations with the NSCBC method, see for instance Kaufmann et al. [16], Guézennec and Poinsot [17] and Pirozzoli and Colonius [18]. The main limitation for the injection of waves comes from the interpretation of the characteristics. They are identified by Poinsot and Lele as the time variation of the waves amplitudes in the linear regime [7], but their exact meaning remains undefined for nonlinear perturbations. Such an interpretation is required to specify the injection of nonlinear waves through the boundaries, especially when nonlinear interactions occur between waves.

In the present paper, one-dimensional non-reflecting characteristic boundary conditions based on nonlinear Riemann invariants are derived. An interpretation of the characteristics, valid for perturbations in the nonlinear regime, is proposed and used to write equations for the injection of large perturbations of acoustic or entropic nature. All developments are performed assuming an ideal, thermodynamically perfect (i.e. with constant specific heats) gas and isobaric entropy fluctuations. This last assumption might appear a bit restrictive but it can be seen as a first illustration of the efficiency of the new formulation with nonlinear perturbations on the boundaries. It is for instance sufficient to investigate the generation of noise caused by the acceleration of such entropy fluctuations through a nozzle [19,20]. The paper is organized as follows. The initial characteristic relations of Thompson [5,6] are first recalled in Section 2. Nonlinear invariants are derived in Section 3 and used to provide a new set of characteristic equations. Relations between the characteristic equations of Thompson and the new set of equations are provided in this section. The new characteristic relations are then exploited in Section 4 to provide expressions for the injection of perturbations in the presence of nonlinear interactions on the boundaries. The influence of the relaxation terms on the behavior of the almost non-reflecting boundary conditions is widely discussed in Section 5 and nonlinear corrective terms are proposed to recover perfectly transparent boundaries for outgoing and ingoing waves. The efficiency of the proposed formulation is demonstrated in Section 6 where simulations are performed for two cases representative of practical configurations. Finally, conclusions and perspectives are given in Section 7.

2. Characteristic relations of Thompson

The one-dimensional Euler equations write, for primitive variables

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = 0 \tag{1}$$

with

$$U = \begin{pmatrix} \rho \\ p \\ u \end{pmatrix}, \qquad A = \begin{pmatrix} u & 0 & \rho \\ 0 & u & \gamma p \\ 0 & \frac{1}{\rho} & u \end{pmatrix} \tag{2}$$

where ρ, p and u are the fluid density, pressure and velocity, respectively, and γ the adiabatic coefficient. The determinant of matrix A is $\det(A) = c^3 M(M^2 - 1)$ with $c = \sqrt{\gamma p/\rho}$ the speed of sound and $M = u/c$ the Mach number. For a flow being not nil ($M \neq 0$) and not sonic ($M \neq \pm 1$), A is diagonalizable and one can write