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A new efficient 3D Discontinuous Galerkin Time Domain (DGTD) method for large and multiscale electromagnetic simulations

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ABSTRACT

A new Discontinuous Galerkin Time Domain (DGTD) method for solving the 3D time dependent Maxwell's equations via the electric field intensity **E** and magnetic flux density **B** fields is proposed for the first time. It uses curl-conforming and divergence-conforming basis functions for **E** and **B**, respectively, with the same order of interpolation. In this way, higher accuracy is achieved at lower memory consumption than the conventional approach based on the field variables **E** and **H**. The centered flux and Riemann solver are both used to treat interfaces with non-conforming meshes, and both explicit Runge–Kutta method and implicit Crank–Nicholson method are implemented for time integration. Numerical examples for realistic cases will be presented to verify that the proposed method is a non-spurious and efficient DGTD scheme.

1. Introduction

The microelectronic industry is facing tough challenges in the reliability of 3-dimensional integrated circuits [1–4]. In 3D IC technology, chips and package are merging closer and closer in a single compact multiscale structure. The cross section of a two-level package based on solder joints is shown in Fig. 1, in which the typical dimensions reveal the multiscale nature of the problem. In this context, Joule heating and vertical interconnects are the two main problems to be solved. For the latter, the potential issues on signal/power integrity (SI/PI) and electromagnetic interference (EMI) should be analyzed in detail. In order to help in this task, efficient and accurate multiscale simulators must be developed. This topic is the focus of the present work.

The conventional Finite-Difference Time-Domain (FDTD) method and Finite-Element Time-Domain (FETD) method have serious difficulties to solve multiscale EM systems. FDTD is a well-known, efficient and simple-to-implement method that uses an orthogonal grid to discretize the structure [5]. It requires high density cells to capture electrically fine structures, thus wasting a large number of unknowns in coarse parts. Another issue of the FDTD method is its conditional stability for solving the time integration, i.e. electrically small structures will lead to a very small (usually non-practical) time step. The FETD method [6] is more flexible in geometry modeling, but requires the inversion of the mass matrix (explicit time integration scheme) or the sum of mass matrix and stiffness matrix (implicit time integration scheme). Although this inversion

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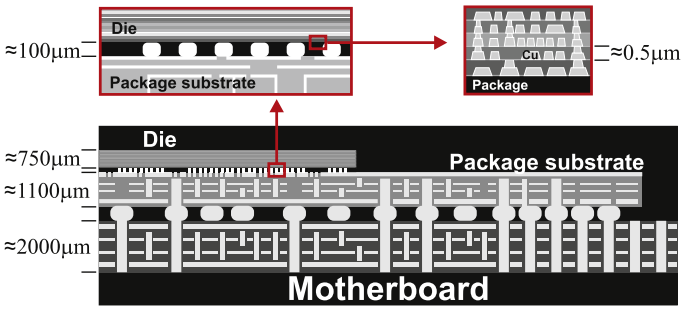

Fig. 1. Multiscale package-to-chip structure.

could be implemented iteratively or approximately [7–9], the computation load could still be heavy when the number of unknowns is large (e.g. multiscale cases, geometry-complex cases).

The discontinuous Galerkin finite-element time-domain (DG-FETD) method is crucial in transient simulations of multiscale electromagnetic systems [10–18]. Its special capability in geometric modeling, by dividing the whole computational domain into several subdomains, transforms a large system into smaller systems with moderate-sized matrices. Another advantage of this method is that each subdomain can be discretized, in both space and time, independently. The numerical fluxes, which communicate fields between adjacent subdomains, are defined by tangential components of electric field **E** and magnetic field **H** on the interfaces [19]; for this reason, the conventional DG-FETD is traditionally based on these two variables, even the FETD method based on the variables **E** and **B** has been studied before for the single-domain continuous Galerkin method [8,20–25]. A correct discretization of these two fields, **E** and **H**, requires curl-conforming basis functions (e.g. the first family of the Nédélec elements) with different orders of interpolation polynomials to suppress spurious solutions [26,27]. For example, in the scheme of $\mathbf{E_nH_{n+1}}$, **E** and **H** use n-th and $(n+1)$-th order of curl-conforming basis functions, respectively. Despite of the higher order of interpolation in one field, the accuracy of the method is dominated by the lower order. Therefore, it will lead to costly memory consumption. For example, in the **E**1**H**2 scheme, the number of unknowns in the **H** field is several times higher than the **E** field.

Here, we propose a DGTD method based on **E** and **B** fields, instead of **E** and **H**. In this way, curl- and divergence-conforming basis functions with the same order of interpolation polynomials are used to discretize the fields, keeping the similar accuracy order and number of unknowns in both families of basis functions. Thus, a much more efficient, accurate, and spurious-mode-free numerical method is developed. Formulations for a single domain are first shown, with results in frequency and time domains. Then, multiple-domain formulations are presented and validated in transient responses. Finally, realistic cases show the efficacy of the proposed method.

2. Governing equations

2.1. Topological laws

The first order Maxwell's equations for electromagnetic fields due to the imposed current densities are considered:

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}_c - \mathbf{J}_s \tag{1}$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} - \mathbf{M}_c - \mathbf{M}_s \tag{2}$$

$$\nabla \cdot \mathbf{D} = \rho_e \tag{3}$$

$$\nabla \cdot \mathbf{B} = \rho_m \tag{4}$$

where **E** and **H** are the electric and magnetic field intensities, respectively; **D** and **B** are the electric and magnetic flux densities, respectively; $\mathbf{J}_c$ and $\mathbf{M}_c$ are the electric and magnetic conduction current densities, respectively; $\mathbf{J}_s$ and $\mathbf{M}_s$ are the imposed electric and magnetic current density sources, respectively; and ρ_e and ρ_m are electric and magnetic charge densities, respectively. Note that we keep magnetic sources $\mathbf{M}_c$, $\mathbf{M}_s$ and ρ_m in these equations for symmetry, even though they are zero in general.

2.2. Constitutive laws

Material properties define the relations between field intensities and flux densities as follows:

$$\mathbf{D} = \epsilon \mathbf{E} \tag{5}$$

$$\mathbf{B} = \mu \mathbf{H} \tag{6}$$