Journal of Computational Physics 283 (2015) 436–452

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

Coupled Vlasov and two-fluid codes on GPUs

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ARTICLE INFO

Article history:
Received 20 June 2014
Received in revised form 28 November 2014
Accepted 1 December 2014
Available online 15 December 2014

Keywords:
Coupling
Vlasov equation
Multifluid
GPU
Whistler waves
Reconnection
GEM

ABSTRACT

We present a way to combine Vlasov and two-fluid codes for the simulation of a collisionless plasma in large domains while keeping full information on the velocity distribution in localised areas of interest. This is made possible by solving the full Vlasov equation in one region while the remaining area is treated by a 5-moment two-fluid code. In such a treatment, the main challenge of coupling kinetic and fluid descriptions is the interchange of physically correct boundary conditions between the different plasma models. In contrast to other treatments, we do not rely on any specific form of the distribution function, e.g. a Maxwellian type. Instead, we combine an extrapolation of the distribution function and a correction of the moments based on the fluid data. Thus, throughout the simulation both codes provide the necessary boundary conditions for each other. A speed-up factor of around 10 is achieved by using GPUs for the computationally expensive solution of the Vlasov equation. Additional major savings are obtained due to the coupling where the amount of savings roughly corresponds to the fraction of the domain where the kinetic equations are solved. The coupled codes were then tested on the propagation of whistler waves and on the GEM reconnection challenge.

1. Introduction

For many relevant problems in plasma physics, the plasma can be considered nearly collisionless. In these cases, the Vlasov–Maxwell system is sufficient for a complete description of the physically important phenomena. Nevertheless, it is close to impossible to numerically solve the Vlasov equation for global problems on fluid scales due to the vast amount of memory needed to resolve a 6-dimensional phase-space. However, in many situations a complete kinetic description is not necessary and computationally much less expensive fluid models based on the moments of the distribution function are sufficient. In addition, plasmas are typical multi-scale phenomena. Often there are only localised and small regions, where a kinetic description is needed, whereas the rest of the domain can be described by means of a fluid model. This separation of scales can be found in many typical problems of plasma physics, such as reconnection in the magnetotail of the earth, solar flares or typical phenomena in tokamaks. To determine what amount of computational savings coupling techniques might provide, a rough estimation of the numerical costs can be made on the basis of spatial scales. To do this, both the computational effort for the fluid region and the overhead for the coupling itself can be assumed as negligible compared to the costs of the kinetic part of the simulation. Thus the fraction of the domain where kinetic effects occur can indicate how much computational time might be saved. This fraction can be estimated on the basis of typical plasma parameters.

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http://dx.doi.org/10.1016/j.jcp.2014.12.016

The most relevant scales are determined by the ion skin depth, below which the dynamics of ions and electrons decouple, and the ion gyro radius, which is especially important in the context of collisionless plasmas.

In the magnetotail of the earth, kinetic effects are important in and around the current sheet, where reconnection takes place. In an example of a typical setup for this problem given by Birn et al. [1], the current sheet—and thus the kinetic region—covers around 10% of the computational domain. In a typical tokamak with an overall size $\sim$1 m, the ion skin depth is $\sim$5 cm (see Woods [2]). The ion gyro radius is even one order of magnitude smaller. Thus, taking into account the rotationally symmetric geometry of a tokamak and the magnetic field therein, the fraction of the domain where localised kinetic effects as reconnection might take place is likely to scale with the fraction of 5 cm/1 m and can thus be expected to be only 5% of the domain, to give a very rough estimation of the order of magnitude. In a solar flare, the ratio is even smaller: A typical size of $\sim 10^7$ m compares to an ion skin depth of about 10 m (see for example Ramesh et al. [3]). We stress that this is a rough estimate assuming e.g. that there are isolated reconnection sites and no turbulent reconnection events where the turbulence scales meet the kinetic scales. In this situation, the fraction of the simulation domain occupied by the kinetic region would be substantially larger.

If the kinetic regions are known, this additional information might be used to reduce the computational costs drastically by restricting the expensive kinetic simulation to this smaller area and using a suitable fluid model for the rest of the domain. There have been several approaches to exploit this ansatz in numerical schemes. In the works of Degond et al. [4], Dellacherie [5], Goudon et al. [6], Klar et al. [7], and Le Tallec and Mallinger [8] methods for coupling the Boltzmann equation to some kind of fluid model like the Navier–Stokes or Euler equations are addressed. Sugiyama and Kusano [9] and especially Daldorff et al. [10] describe the coupling of PIC and MHD models, Markidis et al. [11] show a way to combine PIC and fluid models, and Kolobov and Arslanbekov [12] describe the transition from neutral gas models to models of weakly ionised plasmas with respect to coupling kinetic and fluid equations. Schulze et al. [13] use coupling techniques in the context of epitaxial growth.

Most of these schemes are based on the strong assumption that the distribution function is close to Maxwellian in the vicinity of the coupling border and use this approximation to generate boundary conditions for the kinetic region. In other approaches, an artificial field is introduced that expresses the deviation from a Maxwellian and is coupled to the underlying equations [4], or the fluxes at the boundary are used for the coupling [8].

This paper presents a different mechanism for coupling the Vlasov equation with a five-moment model and the full Maxwell equations. Our method is easy to implement and does not rely on any specific assumptions with respect to the distribution function near the coupling border. In addition, no extensive transition region between the kinetic and fluid domains is required.

The rest of this paper is organised as follows: After the physical models have been described in Section 2, the numerical schemes are presented in Section 3. The coupling procedure is explained in Section 4. Technical details on CUDA graphics cards and their use on the Vlasov code as well as parallel programming issues are discussed in Section 5. Finally, in Sections 6 and 7, first numerical results are presented and discussed.

2. Physical models

Consider a non-relativistic plasma consisting of different particle species s. The Vlasov equation

$$\partial_t f_s + \mathbf{v} \cdot \nabla_{\mathbf{x}} f_s + \frac{q_s}{m_s}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \nabla_{\mathbf{v}} f_s = 0 \tag{1}$$

describes the time evolution of the phase-space density $f_s(\mathbf{x}, \mathbf{v}, t)$, where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic field, q_s is the charge, and m_s is the mass per particle. By calculating the moments of f_s with respect to $\mathbf{v}$, we can derive important physical quantities describing the plasma:

the mass density $$\rho_s = m_s \int f_s \mathrm{d}^3 v, \tag{2}$$

the momentum density $$\mathbf{u}_s = m_s \int \mathbf{v} f_s \mathrm{d}^3 v, \tag{3}$$

and the energy density $$\mathcal{E}_s = \frac{m_s}{2} \int \mathbf{v}^2 f_s \mathrm{d}^3 v. \tag{4}$$

Their time dependence can be calculated from the Vlasov equation (1) and is given by:

$$\partial_t \rho_s = -\nabla \cdot \mathbf{u}_s \tag{5}$$

$$\partial_t \mathbf{u}_s = -\nabla \cdot \left(\frac{\mathbf{u}_s \otimes \mathbf{u}_s}{\rho_s} + \mathbb{P}_s \right) + \frac{q_s}{m_s}(\rho_s \mathbf{E} + \mathbf{u}_s \times \mathbf{B}_s) \tag{6}$$

$$\partial_t \mathcal{E}_s = -\nabla \cdot \left(\frac{\mathcal{E}_s \mathbb{1} + \mathbb{P}_s}{\rho_s} \mathbf{u}_s \right) - \nabla \cdot \mathbf{Q}_s + \frac{q_s}{m_s} \mathbf{u}_s \cdot \mathbf{E}, \tag{7}$$

where $\mathbb{P}_s$ is the pressure tensor and $\mathbf{Q}_s$ is the heat flux tensor. These quantities must be provided in order to close the system.