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Second order finite volume scheme for Maxwell's equations with discontinuous electromagnetic properties on unstructured meshes

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ABSTRACT

This paper presents a second order finite volume scheme for numerical solution of Maxwell's equations with discontinuous dielectric permittivity and magnetic permeability on unstructured meshes. The scheme is based on Godunov scheme and employs approaches of Van Leer and Lax–Wendroff to increase the order of approximation. To keep the second order of approximation near dielectric permittivity and magnetic permeability discontinuities a novel technique for gradient calculation and limitation is applied near discontinuities. Results of test computations for problems with linear and curvilinear discontinuities confirm second order of approximation. The scheme was applied to modelling propagation of electromagnetic waves inside photonic crystal waveguides with a bend.

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1. Introduction

Time dependent Maxwell's equations are the fundamental equations of electrodynamics. They describe propagation and diffraction of electromagnetic waves. For many problems analytical solutions are not available, and as a result various numerical methods were developed [1].

Finite-difference time-domain (FDTD) method was suggested by Yee in [2]. This method employs structured staggered cartesian grids. For the case of constant electromagnetic properties it has second order of approximation in space and time. This method is simple and can provide satisfactory results for a number of problems [3]. However it cannot adequately represent curvilinear boundaries of computational region and boundaries between subregions with different electromagnetic properties. As a result FDTD method loses precision for curvilinear boundaries.

Finite-volume time-domain method (FVTD) can use structured non-cartesian as well as unstructured grids and can better represent curvilinear boundaries. A number of finite volume schemes were developed. Shankar, Hall and Mohammadian proposed a cell-centered scheme in [4]. Madsen and Ziolkowski in [5] suggested staggered finite volume schemes. Remaki presented a cell centered leap frog scheme in [6]. Deore and Chatterjee developed a cell – vertex finite volume scheme in [7]. Edelvik and Ledfelt developed a hybrid solver that allows to combine FVTD and FDTD in [8]. Fumeaux, Baumann, Almpanis, Li and Vahldieck in [9] presented a survey outlining current limitations and challenges for FVTD.

Arguably the major challenge for developing FVTD and FDTD schemes is preservation of high order of approximation for the case of discontinuous electromagnetic properties. In [10] several ways of dielectric permittivity properties smoothing

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were considered. This approach allowed to achieve better precision for some test cases but not the second order of convergence. Dridi, Hesthaven and Ditkowski suggested modified FDTD scheme [11] that preserved second order of convergence.

In [12] Lebedev, Fedoruk and Shtyrina developed an approach based on approximating Maxwell's equations written using continuous variables for dielectric permittivity discontinuity along coordinate line. In [13] Ismagilov extended this approach to curvilinear discontinuities. This led to better accuracy but again not the second order of approximation. In [14] Shi suggested a Runge–Kutta monotonic upstream-centered scheme for conservation laws (MUSCL scheme) with new variable reconstruction approach using least squares method but second order of convergence for discontinuous electromagnetic properties was not demonstrated.

In this paper we present a finite volume scheme that is second order accurate for discontinuous dielectric permittivity and magnetic permeability. The scheme is based on the same second order finite volume scheme as [12] and combines approaches of Godunov [15], Van Leer [16], and Lax and Wendroff [17]. The key feature of the scheme is calculation and limitation of derivatives in cells adjacent to dielectric permittivity and magnetic permeability discontinuities. Stencils for gradient calculation are chosen so that they do not cross discontinuities of dielectric permittivity and magnetic permeability. Limitation is performed either independently for regions with different values of dielectric permittivity and magnetic permeability or using continuous variables on the discontinuity. Computational results confirm second order of approximation of the scheme built for two-dimensional transverse electric (TE) and transverse magnetic (TM) systems of Maxwell's equations for linear and curvilinear discontinuities for independent and continuous variables gradient limitations. Scheme was applied to modelling electromagnetic wave propagation in photonic crystal waveguide with a bend [18].

2. Maxwell's equations

The system of Maxwell's equations in dimensionless form in the absence of charges and currents can be written as

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H}, \qquad \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}, \tag{1}$$

$$\nabla \cdot \mathbf{D} = 0, \qquad \nabla \cdot \mathbf{B} = 0, \tag{2}$$

where $\mathbf{E}$ – electric field, $\mathbf{H}$ – magnetic field, $\mathbf{D} = \varepsilon \mathbf{E}$ – electric flux density, $\mathbf{B} = \mu \mathbf{H}$ – magnetic flux density, ε – dielectric permittivity, μ – magnetic permeability [19].

If media electromagnetic properties ε and μ are discontinuous across a surface, electric and magnetic fields and flux densities can also be discontinuous on the same surface. However by integrating (1) and (2) it can be shown that tangential components of $\mathbf{E}$ and $\mathbf{H}$ and normal components of $\mathbf{D}$ and $\mathbf{B}$ are continuous on the discontinuity of electromagnetic properties [19]. Therefore at a point with normal $\mathbf{n}$ on the discontinuity the following quantities are continuous

$$\begin{aligned} &\mathbf{n} \times \mathbf{H}, \qquad \mathbf{n} \times \mathbf{E}, \\ &\mathbf{n} \cdot \mathbf{D}, \qquad \mathbf{n} \cdot \mathbf{B}. \end{aligned} \tag{3}$$

By taking the divergence of (1) it can be shown that if (2) holds for initial conditions it will be true for solution of (1) at all times [19]. Therefore we consider only time dependent part of Maxwell's equations (1) that can be written in scalar form as

$$\begin{aligned} \frac{\partial D_1}{\partial t} &= \frac{\partial H_3}{\partial x_2} - \frac{\partial H_2}{\partial x_3}, & \frac{\partial B_1}{\partial t} &= \frac{\partial E_2}{\partial x_3} - \frac{\partial E_3}{\partial x_2}, \\ \frac{\partial D_2}{\partial t} &= \frac{\partial H_1}{\partial x_3} - \frac{\partial H_3}{\partial x_1}, & \frac{\partial B_2}{\partial t} &= \frac{\partial E_3}{\partial x_1} - \frac{\partial E_1}{\partial x_3}, \\ \frac{\partial D_3}{\partial t} &= \frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2}, & \frac{\partial B_3}{\partial t} &= \frac{\partial E_1}{\partial x_2} - \frac{\partial E_2}{\partial x_1}. \end{aligned} \tag{4}$$

2.1. Reduction to two dimensions

Assuming that all the fields are independent of the coordinate x_3, x_3 – derivatives in time dependent part of Maxwell's equations (4) vanish, and the resulting two-dimensional system of equations may be decoupled into two systems one for D_3, B_1, B_2

$$\frac{\partial D_3}{\partial t} = \frac{\partial H_2}{\partial x_1} - \frac{\partial H_1}{\partial x_2}, \qquad \frac{\partial B_1}{\partial t} = -\frac{\partial E_3}{\partial x_2}, \qquad \frac{\partial B_2}{\partial t} = \frac{\partial E_3}{\partial x_1}, \tag{5}$$

and another one for D_1, D_2, B_3

$$\frac{\partial D_1}{\partial t} = \frac{\partial H_3}{\partial x_2}, \qquad \frac{\partial D_2}{\partial t} = -\frac{\partial H_3}{\partial x_1}, \qquad \frac{\partial B_3}{\partial t} = \frac{\partial E_1}{\partial x_2} - \frac{\partial E_2}{\partial x_1}, \tag{6}$$

respectively termed TM system and TE system. Both systems can be written in vector form as

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