Journal of Computational Physics 282 (2015) 47–61

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A hybrid immersed boundary and immersed interface method for electrohydrodynamic simulations

Wei-Fan Hu [a], Ming-Chih Lai [b,*], Yuan-Nan Young [c]

[a] Department of Applied Mathematics, National Chiao Tung University, 1001, Ta Hsueh Road, Hsinchu 300, Taiwan
[b] Center of Mathematical Modeling and Scientific Computing & Department of Applied Mathematics, National Chiao Tung University, 1001, Ta Hsueh Road, Hsinchu 300, Taiwan
[c] Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, NJ 07102, USA

ARTICLE INFO

Article history:
Received 28 February 2014
Received in revised form 15 August 2014
Accepted 5 November 2014
Available online 11 November 2014

Keywords:
Immersed boundary method
Immersed interface method
Elliptic interface problem
Electrohydrodynamics
Leaky dielectric model
Navier–Stokes equations

ABSTRACT

In this paper, we develop a hybrid immersed boundary (IB) and immersed interface method (IIM) to simulate the dynamics of a drop under an electric field in Navier–Stokes flows. Within the leaky dielectric framework with piecewise constant electric properties in each fluid, the electric stress can be treated as an interfacial force on the drop interface. Thus, both the electric and capillary forces can be formulated in a unified immersed boundary framework. The electric potential satisfies a Laplace equation which is solved numerically by an augmented immersed interface method which incorporates the jump conditions naturally along the normal direction. The incompressible Navier–Stokes equations for the fluids are solved using a projection method on a staggered MAC grid and the potential is solved at the cell center. The interface is tracked in a Lagrangian manner with mesh control by adding an artificial tangential velocity to transport the Lagrangian markers to ensure that the spacing between markers is uniform throughout the computations. A series of numerical tests for the present scheme have been conducted to illustrate the accuracy and applicability of the method. We first compute the potential and its gradient (electric field) to perform the accuracy check for the present augmented IIM. We then check the convergence of the interfacial electric force and the fluid variables. We further run a series of simulations with different permittivity and conductivity ratios and compare with the results obtained by the small deformation theory and other numerical results in literature. In addition, we also study the electric effect for a drop under shear flow.

1. Introduction

The study of hydrodynamics driven by an electric field (electrohydrodynamics) has many industrial applications in microfluidic systems [22,33]. In particular, a weakly conducting (leaky dielectric) drop suspended in another leaky dielectric fluid under an electric field (as depicted in Fig. 1) has been extensively studied from different perspectives. G.I. Taylor [27] concluded that the equilibrium drop shape can be explained by balancing the viscous stress with the electric stress on the drop interface as long as he took into account the induced surface charges due to the mismatch in electric conductivity and permittivity between the two fluids. Furthermore, it is found that drop can be deformed into either a prolate or an oblate

* Corresponding author.
E-mail addresses: weifanhu.am95g@g2.nctu.edu.tw (W.-F. Hu), mclai@math.nctu.edu.tw (M.-C. Lai).

http://dx.doi.org/10.1016/j.jcp.2014.11.005

equilibrium shape with circulatory flows inside the drop [20]. The drop deformation and flow patterns mainly depend on the electrical properties of the fluid system. A comprehensive review of further theoretical developments can be found in [24]. Most theoretical models are limited to small deformation from a spherical drop under moderate electric fields, and such limitation leads to quantitative discrepancies reported in [29]. Recently a spheroidal model was used to study the large electrodeformation of a leaky dielectric drop (see [21] and references therein). Results from these models agree well with experiments up to moderate electric capillary number. However, these spheroidal models still cannot predict the drop deformation under large electric field strengths.

Thus, there has been a continuing interest in using numerical methods to simulate the electrohydrodynamics of a viscous drop under an electric field. Numerous works on the numerical simulations of drop deformations under leaky dielectric theory are reported in the literature. Based on how the interface is treated, these works can be categorized into the front tracking method [11,30], level set method [4,31], phase field method [18,35], and the volume-of-fluid method [9,32]. Other numerical approaches include lattice Boltzmann method [36] and boundary integral method [13,25], just to name a few.

In this paper, a hybrid immersed boundary (IB) and immersed interface method (IIM) are developed to simulate the dynamics of a leaky dielectric drop under an electric field in Navier–Stokes fluids. The immersed boundary method used to solve the fluid equations is similar in spirit to the front-tracking method as in [11,30]. Other recent immersed boundary methods to study particle–particle interactions in electrohydrodynamics can be found in [3,10], in particular, the former one used the available immersed interface method solver [17] to solve the potential equation but the applications and the electric force calculations are different from the present study.

The major contributions and significant differences of our work from previous works [4,3,11,10,18,30–32,35] are as follows. Firstly, the Maxwell stress tensor arising from the electric effect is cast as an interfacial electric force (the jump of Maxwell tensor across the interface) rather than the volume force in the equations so that the capillary and electric interfacial forces can be formulated in a unified immersed boundary framework. As can be seen in the next section (Section 2), the present approach avoids applying the divergence operator to the Maxwell tensor. This is particularly important since the tensor is discontinuous across the interface due to different permittivity and conductivity.

Secondly, a sharp immersed interface method is used to compute the potential and its gradient. One-sided interpolations to the interface from either side are used to obtain the Maxwell stress tensor (thus the interfacial electric force) more accurately. In most aforementioned works in literature, the electric potential equation Eq. (6) is solved numerically by smoothing the piecewise conductivity σ over a narrow region using the harmonic means without any special treatments near the interface. However, according to the jump condition Eq. (7), one can immediately see that $\nabla\phi$ has nonzero jump across the interface. Thus, by the above smoothing numerical method, the electric field $\mathbf{E} = -\nabla\phi$ will have $O(1)$ error near the interface. Furthermore, the smoothing of permittivity ε also causes $O(1)$ error near the interface. The computations of $\nabla\varepsilon$ and $\nabla\cdot(\varepsilon\mathbf{E})$ both have $O(1/h)$ error near the interface. As a result, the direct calculation of volume electric force Eq. (9) is not accurate near the interface. Here, instead of computing the volume force, we compute the interfacial electric force in a more accurate manner. We are also able to demonstrate the numerical convergence of this interfacial force. A more rigorous theoretical analysis on the accuracy of finite difference schemes for elliptic interface problems can be found in [2].

The paper is organized as follows. The governing equations for the electrohydrodynamics within the leaky dielectric framework are presented in Section 2. A simple version of augmented immersed interface method for solving piecewise elliptic interface problem is introduced and tested in Section 3. A hybrid numerical algorithm of immersed boundary and immersed interface methods for solving the electrohydrodynamics equations is outlined in Section 4. The numerical results consisting of convergence check and drop deformations under DC electric fields with different permittivities and conductivities are studied in detail in Section 5. Some concluding remarks and brief discussion on future directions are given in Section 6.

2. Governing equations of electrohydrodynamics

In this paper, we consider a leaky dielectric drop (Ω^-) suspended in another immiscible leaky dielectric fluid (Ω^+) under a DC (direct current) electric field $\mathbf{E}_\infty$ as depicted in Fig. 1. The governing equations consist of two-dimensional incompressible Navier–Stokes equations with surface tension and electric forces. We further assume that both the density and viscosity are identical for the drop and suspended fluid as we concentrate more on the electro-deformation of the drop due to different electric conductivity and permittivity. The drop is neutrally buoyant in the fluid domain Ω and the gravitational force is neglected. The fluid interface Σ is represented by a Lagrangian parametric form $\mathbf{X}(s,t) = (X(s,t), Y(s,t))$, $0 \le s \le 2\pi$, where s is the parameter of the initial configuration of the interface. Under the immersed boundary formulation, this two-fluid system is cast as a single fluid with variable physical properties in a single domain $\Omega = \Omega^- \cup \Omega^+$

$$\rho(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}) + \nabla p = \mu\Delta\mathbf{u} + \mathbf{f}_C + \mathbf{f}_E \quad \text{in } \Omega, \tag{1}$$

$$\nabla\cdot\mathbf{u} = 0 \quad \text{in } \Omega, \tag{2}$$

$$\frac{\partial \mathbf{X}}{\partial t}(s,t) = \mathbf{U}(s,t) = \int_\Omega \mathbf{u}(\mathbf{x},t)\delta\big(\mathbf{x} - \mathbf{X}(s,t)\big)\,d\mathbf{x} \quad \text{on } \Sigma. \tag{3}$$