Journal of Computational Physics 282 (2015) 148–156

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A vector potential implementation for smoothed particle magnetohydrodynamics

Federico A. Stasyszyn *, Detlef Elstner

Leibniz-Institut für Astrophysik Potsdam (AIP), An der Sternwarte 16, 14482, Potsdam, Germany

ARTICLE INFO

Article history:
Received 2 May 2014
Received in revised form 7 November 2014
Accepted 10 November 2014
Available online 15 November 2014

Keywords:
Magnetohydrodynamics
Astrophysics
Smooth particle hydrodynamics

ABSTRACT

The development of smooth particle magnetohydrodynamic (SPMHD) has significantly improved the simulation of complex astrophysical processes. However, the preservation the solenoidality of the magnetic field is still a severe problem for the MHD. A formulation of the induction equation with a vector potential would solve the problem. Unfortunately all previous attempts suffered from instabilities. In the present work, we evolve the vector potential in the Coulomb gauge and smooth the derived magnetic field for usage in the momentum equation. With this implementation we could reproduce classical test cases in a stable way. A simple test case demonstrates the possible failure of widely used direct integration of the magnetic field, even with the usage of a divergence cleaning method.

1. Introduction

In recent years, not only the presence but the morphology of magnetic fields in galaxies has been determined [2]. It represents a huge scientific challenge, as this is a new opportunity to understand how the magnetic field is related to the astrophysical hosts, their history and properties.

A possible explanation for the magnetic field amplification is the action of a dynamo driven by turbulence and large scale gas motions [23,1,19], where the unknown initial seed field is washed out by the turbulent character of the flow. Numerical simulations of evolving galaxies should help to understand the main properties of the magnetic field amplification with the observed morphology.

The success of cosmological simulations using SPH methods motivates the application of that technique also for the MHD case [10]. The direct implementation of the induction equation with the magnetic field unfortunately suffers from the preservation of solenoidality. The artificial growth of $\nabla \cdot \mathbf{B}$ in these schemes is usually reduced by a more or less artificial cleaning of **B**. The direct integration of **B** with or without cleaning may lead to unrealistic numerical growth of the magnetic field as it occurs in the example described in Section 3 or in Kotarba et al. [13]. There is no $\nabla \cdot \mathbf{B} = 0$ preserving scheme known for SPMHD integrating the magnetic field **B** directly from the induction equation. Changing the integration variable from the magnetic field to the vector potential **A** with $\mathbf{B} = \nabla \times \mathbf{A}$ solves the problem in a natural way. A vector potential formulation in SPH was previous studied in detail by Price [17]. The implementation was working for one and two dimensional problems, but failed in three dimensions.

* Corresponding author.
E-mail addresses: fstasyszyn@aip.de (F.A. Stasyszyn), delstner@aip.de (D. Elstner).

http://dx.doi.org/10.1016/j.jcp.2014.11.011
0021-9991/

In the following sections we present an application of the vector potential in SPMHD, which overcomes the previous problems. In Section 2 we shortly describe our implementation, followed by the analysis of some test cases in Section 3. Finally we discuss possible implications in Section 4 and we present our conclusions in Section 5.

2. SPH implementation

Throughout this work we will use the SPMHD version of Gadget-3 [20], where the ideal MHD is solved following the induction equation in the form

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} = (\mathbf{B}\cdot\nabla)\mathbf{v} - \mathbf{B}(\nabla\cdot\mathbf{v}) \tag{1}$$

in which, we assume the $\nabla\cdot\mathbf{B} = 0$ constraint is valid, by taking special care on reducing it [20,22].

Price [17] studied carefully the possible SPH vector potential formulation, we follow it and use it as a starting point. The definition of the magnetic field and the evolution of the vector potential can be summarized as follows:

$$\mathbf{B} = \nabla\times\mathbf{A} \tag{2}$$

$$\frac{\mathrm{d}\mathbf{A}}{\mathrm{d}t} = \mathbf{v}\times\nabla\times\mathbf{A} + (\mathbf{v}\cdot\nabla)\mathbf{A} - \nabla\phi \tag{3}$$

where ϕ is an arbitrary scalar representing the freedom to choose a special gauge. There is the freedom of choosing different gauges for each time step if desired to improve the numerics, but keeping track of a proper ϕ evolution of a given particular gauge.

In tensor form the components of Eq. (3) simplify to

$$\frac{\mathrm{d}A_i}{\mathrm{d}t} = v^j\frac{\partial A_j}{\partial x^i} - \frac{\partial\phi}{\partial x^i} \tag{4}$$

where i, j are component indexes and summation over double indices is used.

In the SPH framework this equation is written as follows,

$$\frac{\mathrm{d}A_a^i}{\mathrm{d}t} = \frac{f_a}{\rho_a}\left[\sum_{b=1}^{N} -m_b(\phi_{ab}^i - v_a^j A_{ab}^j)\partial W_{ab}^i\right] \tag{5}$$

where a, b are particle indexes, f_a is the correction factor that arises from the use of variable smoothing lengths, the A_{ab}^j is the difference between the potential of neighboring particles and ∂W_{ab}^i is the kernel gradient operator between particles (for more details refer to Dolag and Stasyszyn [10]).

As we mentioned before, the gauge choice does not manifest in the magnetic field, but in the evolution of the vector potential. For example, if we use the *Coulomb* gauge, which means $\nabla\cdot\mathbf{A} = 0$ for all points of space and time, we have to take care of fulfilling this requirement. Therefore, we face a similar problem as keeping $\nabla\cdot\mathbf{B} = 0$, that has already been extensively studied [20,22]. We take a similar approach, using a cleaning scheme [9] originally thought to lower the $\nabla\cdot\mathbf{B}$ errors, but applied to $\mathbf{A}$ in order to ensure $\nabla\cdot\mathbf{A} = 0$. The solution of the problem is equivalent to choose a modified *pseudo-Lorenz* or *velocity* gauge [12,6], with an additional damping term. Note, that keeping $\nabla\cdot\mathbf{A} = 0$ will also simplify the calculation of the diffusion terms for the non-ideal MHD equations. The evolution of the gauge is achieved through following equations

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = -c_h^2\,\nabla\cdot\mathbf{A} - c_h\frac{\phi}{h} - \frac{\nabla\cdot\mathbf{v}\,\phi}{2} \tag{6}$$

where c_h is the characteristic signal velocity, h is the smoothing length and we add the final term, introduced by Tricco and Price [22] that takes into account compression or expansion of the fluid. Tricco and Price [22] found that this additional term, improves conservation of energy and in particular for the divergence cleaning is crucial the symmetrization of the SPH operators. In our case we use a "differential" non-symmetric SPH operator and we do not apply any *limiter* as in Stasyszyn et al. [20], and seems sufficient to achieve stability. However, when coupling the energy evolution using a symmetric operator can improve the energy conservation.

Therefore the gauge evolution in SPH form writes as Eq. (6), and the differential operators takes the form for the divergence case as:

$$\nabla\cdot\mathbf{A} = \frac{f_a}{\rho_a}\left[\sum_{b=1}^{N} m_b A_{ab}^i\partial W_{ab}^i\right] \tag{7}$$