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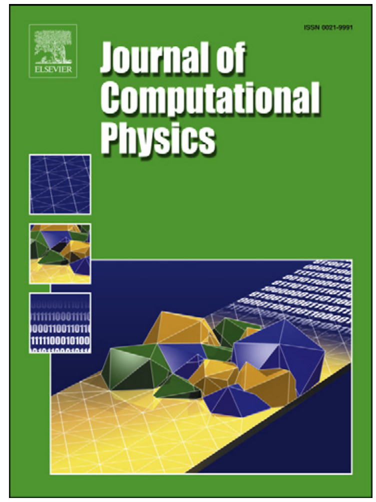

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Numerical solving an equation for fractional powers of elliptic operators

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Abstract

An equation for a fractional power of the second-order elliptic operator is considered. It is solved numerically using a time-dependent problem for a pseudo-parabolic equation. For the auxiliary Cauchy problem, the standard two-level schemes are applied. Stability conditions are obtained for the fully discrete schemes under the consideration. The numerical results are presented for a model two-dimensional problem with a fractional power of an elliptic operator. The dependence of accuracy on grids in time and in space is studied.

Keywords: Elliptic operator, fractional power of an operator, two-level scheme, stability of fully discrete schemes, finite element approximations
2010 MSC: 35R11, 65F60, 65M06, 65N22

1. Introduction

Nowadays, non-local applied mathematical models based on the use of fractional derivatives in time and space are actively discussed [1, 2, 3]. An interesting example is an equation for a fractional power of an elliptic operator. For example, suppose that in a bounded domain Ω on the set of functions $u(\boldsymbol{x}) = 0, \ \boldsymbol{x} \in \partial\Omega$, there is defined the operator A: $Au = -\triangle u, \ \boldsymbol{x} \in \Omega$. We seek the solution of the equation

$$A^\alpha u = f$$

for the given $f(\boldsymbol{x}), \ \boldsymbol{x} \in \Omega$ with $0 < \alpha < 1$.

Different approaches are employed to solve numerically such problems. The simplest variant is associated with the explicit construction of the solution using the known eigenvalues and eigenfunctions of the elliptic operator with diagonalization of the corresponding matrix [4, 5, 6]. Unfortunately, this approach demonstrates too high computational complexity for multidimensional problems.

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