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A spectral unaveraged algorithm for free electron laser simulations

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ABSTRACT

We propose and discuss a numerical method to model electromagnetic emission from the oscillating relativistic charged particles and its coherent amplification. The developed technique is well suited for free electron laser simulations, but it may also be useful for a wider range of physical problems involving resonant field–particles interactions. The algorithm integrates the unaveraged coupled equations for the particles and the electromagnetic fields in a discrete spectral domain. Using this algorithm, it is possible to perform full three-dimensional or axisymmetric simulations of short-wavelength amplification. In this paper we describe the method, its implementation, and we present examples of free electron laser simulations comparing the results with the ones provided by commonly known free electron laser codes.

1. Introduction

When relativistic charged particles propagate through a periodically modulated field, they oscillate and emit a radiation with a shorter wavelength due to the Doppler effect (see Fig. 1). This is the principle of Synchrotron Radiation (SR) sources, where beams of accelerated electrons are deviated by a wiggling magnetic field in order to produce bright X-rays. In state-of-the-art sources, the electrons are produced by a linear accelerator and are wiggled in a series of alternating permanent magnets known as an *undulator*. A particularly interesting case is that of free electron lasers (FEL), where the SR source operates in a coherent regime. In these devices, the emitted X-rays have a resonant interaction with the electrons, and they are efficiently amplified via Stimulated Compton Scattering (see [1,2] for recent reviews of FELs). Through this process, X-ray free electron lasers (XFELs) can currently reach the multi-GW level in peak-power, at the angström wavelengths [3].

In order to describe the physics of free electron lasers, major theoretical results were obtained both in the quantum and classical approaches [4–7], and appropriate numerical tools were developed [8]. To simulate the particles–field interaction, one may use an approach similar to the method adopted in the particle-in-cell (PIC) schemes [9]. In this approach, the Maxwell equations are solved on a spatial grid by the finite difference time-domain (FDTD) methods. The electrons are represented by charged macro-particles and the particles–radiation interaction is conveyed by interpolation techniques. In the case of relativistic particles interacting with an X-ray radiation, this requires a grid which accurately resolves the wavelength of the amplified radiation λ_s as well as the electron beam size l_b. In XFELs the ratio l_b/λ_s may be as high as 10^5–10^6, and the direct implementation of such methods results in a large computational load.

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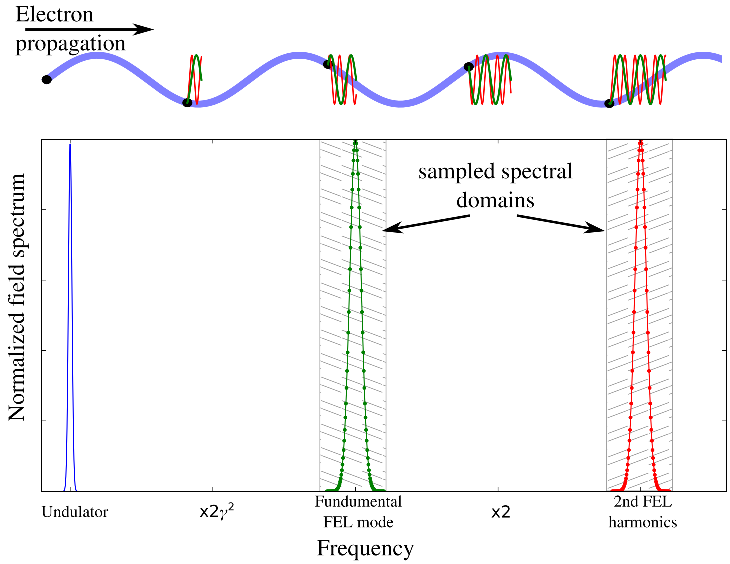

Fig. 1. Schematic representation of the emission from an oscillating relativistic particle, and of its spectrum.

In the dedicated FEL codes this problem is overcome by making a few approximations. One approximation, used in the majority of the mainline FEL codes is the *slowly varying envelope approximation* (SVEA), in which the electromagnetic field is regarded as a wave, $\mathbf{E}(\mathbf{r}, t) = \mathrm{Re}\{\hat{\mathbf{E}}(\mathbf{r}, t)\exp[2\pi i(z - ct)/\lambda_s]\}$, where λ_s is the anticipated resonant wavelength [10] and $\hat{\mathbf{E}}$ is assumed to be slowly-varying in space and time [11]. Often SVEA is complemented with the *wiggler-averaged* formulation which assumes that the trajectories of oscillating electrons follow precisely the sinusoidal field of the undulator. This approach allows to consider only slow longitudinal displacements of electrons with respect to the electromagnetic field without solving the full three-dimensional equations of motion. Both SVEA and wiggler-averaging provide a great calculation efficiency for the conventional FEL modeling, however, in the cases, where fast variations of the radiation profile are essential the correct description may require non-averaged approach [12,13].

Due to the growing interest for the unaveraged FEL modeling, a number of approaches were developed for one-dimensional simulations [12,14], and three-dimensional simulations [15,16]. One approach consists in using the multi-frequency model, which is a pseudo-spectral time domain (PSTD) method, where the Maxwell equations are integrated in a predefined Fourier domain [17]. One such model is presented in [18], where the author uses a one-dimensional Fourier decomposition of the fields along the direction of propagation, centered around the resonant frequency. The orthogonality of the Fourier modes in a finite-length periodic interval allows to write separately the equations for each mode, and to integrate them in time along with the equations of motion for the electrons. Similar approach, using Fast Fourier Transform (FFT) technique, was developed in [14] for one-dimensional unaveraged modeling.

In three-dimensional simulations besides the purely FDTD methods the spectral techniques are often combined with FDTD approach, for example in [16], where the split-step integration accounts for diffraction via FFT, and particles–field interaction is resolved with an FDTD Galerkin method. Alternatively, decomposition of the electromagnetic field into the Gauss–Hermite or Gauss–Laguerre modes is also a popular method, which has been used to represent the transverse profile of the radiation [15,19]. The choice of the correct transverse mode can be advantageous in terms of calculation time and accuracy, and it depends on the symmetry of the problem and its boundary conditions.

In this paper we describe a general approach for three-dimensional modeling of FEL simulations with a spectral method. A fully three-dimensional algorithm is based on Cartesian Fourier series, and for the axisymmetric case we use the cylindrical Fourier–Bessel series. The spectral components of the fields are defined on a discrete grid in Fourier space, and the particles currents are projected onto this spectral grid. The Maxwell equations are integrated numerically for each spectral component, and the forces are calculated by summing the Fourier series at the particles positions. The structure of the spectral domain may be chosen rather flexibly, which provides a simple way to capture the harmonics of the emitted field (see Fig. 1). The required spectral resolution for the field can be rather modest, which in some cases allows to run the simulations on a desktop computer. In this communication we focus on the numerical recipe itself and its implementation in the context of the basic FEL interaction.

In the following, we describe the mathematical model (Section 2) and the main aspects of its implementation in the code `PlaRes` (Section 3), and we discuss the choice of the simulation parameters (Section 4). In Section 5 we introduce several FEL simulations which were run with `PlaRes`, and we compare the results with those obtained by three commonly-used FEL codes. Section 6 summarizes the results and gives prospective applications for `PlaRes`.