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Reactive linearized equations of perturbed compressible variables for low-Mach number variable-density flows

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ABSTRACT

A hydrodynamic/acoustic splitting approach is proposed to study noise emitted from reactive variable-density flows. A simulation using the variable-density low-Mach number equations provides a solution of the hydrodynamic motions of the flow (base-flow), and a set of equations for perturbed variables is additionally solved to capture the acoustic motions. A rigorous derivation of these equations for the assumed base-flow is given, which compared to its non-reacting counterpart includes additional terms related to variable-density flows. Two different test cases are presented. First, the Kirchhoff vortex is simulated to highlight instability issues related to constant-density flows. Second, an academic test case for variable-density flows is proposed in the form of a reacting dipole, which is used to underline the stability of the proposed perturbed equations in such a scenario. Various intermediate forms of the derived perturbation equations are juxtaposed and analyzed with respect to their stability for these two test cases, and assumptions made in their derivation are numerically justified.

1. Introduction

The idea of simulating noise by means of a splitting of the hydrodynamic and acoustic effects into two separate simulations was originally devised by Hardin and Pope [8] and shortly afterwards used in calculations of flow over a cavity [9] as well as co-rotating vortices [12]. A few years later, Shen and Sørensen [26] pointed out some inconsistencies in the original formulation, suggested a slight modification, and demonstrated applicability to laminar flow over a circular cylinder [25] as well as turbulent flow over an airfoil [27]. At the same time, Slimon et al. [29] invented the approach of a rigorous Mach number expansion to retain only relevant terms in the perturbed equations, and Slimon et al. [30] validated their approach with simulations of flow past a cylinder and noise emitted from a mixing layer.

Subsequently, Goldstein [7] generalized this splitting theory in a framework known as Goldstein's acoustic analogy, in which he outlines how an arbitrary, non-radiating flow field can be assumed as the base-flow solution. The latter could, for example, consist of the time-averaged solution of the fully compressible Navier–Stokes equations or the non-radiating but unsteady solution of the incompressible Navier–Stokes equations. In a rigorous derivation, he demonstrated how a subtraction of this assumed base-flow from the fully compressible unsteady Navier–Stokes equations leads to transport equations for a new set of perturbed variables that are driven by a new set of generalized stress tensors.

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Seo and Moon [23] investigated the sources of the numerical instabilities of the above methods, leading to the development of the so-called perturbed compressible equations (PCE). A careful analysis of the role that under-resolved perturbed vorticity plays in these stability considerations provided Seo and Moon [24] with the basis to develop the so-called linearized perturbed compressible equations (LPCE).

A slightly different approach with the same target of removing perturbed vorticity from the transport equations for perturbed variables was outlined by Ewert and Schröder [5]. Their work relied on *a priori* filtering techniques to remove this unwanted quantity, resulting in their so-called acoustic perturbation equations (APE). In contrast to the unsteady but incompressible base-flow assumption by Seo and Moon [23,24], their base-flow was assumed to be the time-average of a fully compressible simulation.

Bui et al. [3] later suggested an extension of the APE to reacting flows, namely APE-RF, in combination with large eddy simulations (LES) to analyze the noise emission from a non-premixed flame. A few years later, Bui et al. [2] validated the APE-RF on the DLR-A flame [1,17,28] and analyzed more carefully different noise source contributions to the overall sound pressure level. Again, the time-average of the LES was taken as the base-flow, which essentially introduces the material derivative of the base-flow density as the primary source term.

The goal of the present work is to extend the idea of Seo and Moon [24] to variable-density flows, resulting in the reactive linearized perturbed momentum equations in gradient form (RLPM-G) as a generalized form of the LPCE. This approach allows the direct use of the *unsteady* variable-density low-Mach number equations as the non-radiating base-flow, thereby retaining the material derivative of the hydrodynamic pressure as the primary source term, as suggested by Goldstein [7].

In the various forms of the APE equations, as discussed in Bui et al. [2], several versions with varying associated source terms are proposed, depending on whether the base-flow is constant- or variable-density. The primary advantage of the present approach is therefore as follows. The proposed equations are a generalized and systematically derived set of equations for perturbed variables which do not raise the need of source term modeling, independent of whether the flow under consideration is constant or variable-density.

In Section 2, the governing equations for the fully compressible and the base-flow simulations are presented, followed by a rigorous derivation of the resulting perturbed equations in Section 3. Simplifying assumptions as well as a detailed analysis of instability effects are analyzed in Section 4 for various forms of the perturbed equations. After a brief summary of the numerical implementation in Section 5, the Kirchhoff vortex test case is presented in Section 6, followed by the proposed reacting dipole test case in Section 7. Lastly, in Section 8, conclusions will be drawn.

2. Fully compressible and base-flow equations

2.1. Fully compressible Navier–Stokes equations

The instantaneous equations governing conservation of mass, momentum, and sensible energy are written in tensor notation, with repeated indices denoting summation, as:

$$\partial_t \rho + \partial_j(\rho u_j) = 0 \tag{1a}$$

$$\partial_t(\rho u_i) + \partial_j(\rho u_i u_j) = -\partial_i p + \partial_j \tau_{ji} \tag{1b}$$

$$\partial_t(\rho e_s) + \partial_j(\rho e_s u_j) = -p\partial_j u_j - \partial_j q_j + \Phi + \dot{\omega}_T, \tag{1c}$$

where t denotes time and i the i-direction of the spatial x_i-coordinate, ρ denotes density, u_i is the fluid velocity in the spatial i-direction, p denotes pressure, τ_{ij} is the viscous stress tensor, e_s is the sensible energy, q_i the heat flux vector, $\Phi = \tau_{ij}\partial_j u_i$ abbreviates the dissipation function, and $\dot{\omega}_T$ stands for the chemical heat release. Sensible energy and sensible enthalpy are defined as the following integral expressions:

$$e_s = \int_{T_{\text{ref}}}^{T} c_v \mathrm{d}T, \qquad h_s = \int_{T_{\text{ref}}}^{T} c_p \mathrm{d}T, \tag{2}$$

where c_p and c_v are the coefficients of specific heat at constant pressure and volume, respectively, and temperature is denoted by T. Sensible energy and enthalpy are related via pressure and density:

$$e_s = h_s - \frac{p}{\rho}. \tag{3}$$

The dependent variables also have to satisfy an equation of state; here the ideal gas law is assumed:

$$p = \rho R T \tag{4a}$$

$$R = c_p - c_v, \tag{4b}$$

where R is the specific gas constant.