Journal of Computational Physics 281 (2015) 225–236

Contents lists available at ScienceDirect

Journal of Computational Physics

www.elsevier.com/locate/jcp

A boundary collocation meshfree method for the treatment of Poisson problems with complex morphologies

Soheil Soghrati [a,b,c,∗,1], Weijie Mai [b], Bowen Liang [a], Rudolph G. Buchheit [b,d]

[a] Department of Mechanical and Aerospace Engineering, The Ohio State University, USA
[b] Department of Materials Science and Engineering, The Ohio State University, USA
[c] Simulation Innovation and Modeling Center, USA
[d] Fontana Corrosion Center, USA

A R T I C L E I N F O

Article history:
Received 26 February 2014
Received in revised form 3 October 2014
Accepted 13 October 2014
Available online 22 October 2014

Keywords:
Green's functions
Fundamental solutions
Meshfree
Discrete transformation
Boundary collocation
Pitting corrosion

A B S T R A C T

A new meshfree method based on a discrete transformation of Green's basis functions is introduced to simulate Poisson problems with complex morphologies. The proposed Green's Discrete Transformation Method (GDTM) uses source points that are located along a virtual boundary outside the problem domain to construct the basis functions needed to approximate the field. The optimal number of Green's functions source points and their relative distances with respect to the problem boundaries are evaluated to obtain the best approximation of the partition of unity condition. A discrete transformation technique together with the boundary point collocation method is employed to evaluate the unknown coefficients of the solution series via satisfying the problem boundary conditions. A comprehensive convergence study is presented to investigate the accuracy and convergence rate of the GDTM. We will also demonstrate the application of this meshfree method for simulating the conductive heat transfer in a heterogeneous materials system and the dissolved aluminum ions concentration in the electrolyte solution formed near a passive corrosion pit.

1. Introduction

Despite remarkable advances in both the computational science and computing resources in the past few decades, the analysis of engineering problems and physical phenomena with complex morphologies remains an ongoing challenge. Such complexities are not only due to the heterostructure and intricate morphology of the domain but also can emanate from the transient evolution of its geometry in problems such as the pitting corrosion [1]. In most cases, while conventional mesh-based techniques such as the standard finite element method (FEM) can accurately approximate the governing equations, the labor and computational costs associated with creating/updating the conforming (matching) mesh undermine the efficiency of such methods [2–4]. Furthermore, the quality of the resulting finite element (FE) mesh, i.e., the elements aspect ratio and the level of refinement, has a direct impact on the accuracy of the solution, which may not easily be assured for problems with complex morphologies [5–7]. In moving boundary problems, while adaptive mesh refinement techniques are

∗ Corresponding author at: Department of Mechanical and Aerospace Engineering, The Ohio State University, USA. Tel.: +1 614 292 276.
E-mail address: soghrati.1@osu.edu (S. Soghrati).

1 Assistant professor of Mechanical and Aerospace Engineering and Materials Science and Engineering, The Ohio State University, 201 W. 19th Avenue, Columbus, OH 43210, USA.

http://dx.doi.org/10.1016/j.jcp.2014.10.030
0021-9991/

often implemented to avoid the expensive re-meshing process, creating new elements with proper aspect ratios involves similar challenges [8,9].

Several numerical techniques have been proposed to mitigate the difficulties associated with the construction of conforming meshes needed in the standard FEM. Polygonal/polyhedral elements allow a more flexible algorithm for meshing complex geometries while providing a higher accuracy and less sensitivity on the element aspect ratio compared to Lagrangian elements [10,11]. The Virtual Element Method (VEM) [12,13] combines the advantages of polygonal/polyhedral elements with ideas from mimetic finite difference method [14,15] and has shown to yield satisfactory results using elements with bad aspect ratios. To allow the implementation of FE meshes that are completely independent of the problem morphology, mesh-independent techniques such as the Immersed FEM [16], eXtended/Generalized FEM [17,18], and the Hierarchical Interface-enriched FEM (HIFEM) [19–21] are also introduced. These methods can retrieve an optimal accuracy and convergence rate for problems discretized with non-matching meshes via incorporating appropriate enrichments along the problem boundaries and/or discontinuous phenomena in the domain.

The Boundary Element Method (BEM) [22,23] is another technique that can alleviate the difficulties associated with the implementation of the standard FEM for modeling problems with complex geometries. In BEM, the strong form of a boundary/initial value problem is converted into a boundary integral equation described in terms of Green's basis functions. The approximation of the field thus requires constructing FE meshes that only discretize the domain boundaries, which considerably reduces the complexity of the mesh generation process by replacing a volume mesh with a surface mesh [24]. One of the key disadvantages of the classical BEM is the high computational cost associated with evaluating the boundary integrals in this method, which can be addressed by using more advanced techniques such as the fast multi-pole BEM [24,25] and the pre-corrected fast Fourier transformation method [26].

Meshfree Methods (MMs) completely eliminate the need for creating FE meshes by constructing a global field approximation over either a set of domain points [27,28] or (for linear problems) only along the boundary points [29]. Among widely used MMs, we can mention the smoothed particle hydrodynamics (SPH) [30,31], the method of fundamental solutions (MFS) [32,33] and the exponential basis functions (EBFs) method [29,34,35]. In SPH which is frequently used for simulating fluid problems, a set of kernel functions is transformed into motion equations to evaluate the governing equations [36]. The MFS approximation series is built on a set of boundary points (boundary collocation technique), using fundamental solutions with source points that are created outside the problem domain [37]. Similarly, the EBFs method only requires the discretization of the problem boundaries, but uses a discrete transformation technique to compute the unknown coefficients of the solution series [38]. Reviews of different MMs and their applications to simulate engineering problems are presented in [31,39].

In this manuscript, we introduce a new meshfree method that uses the same discrete transformation as the EBFs method [29,40] but employs Green's basis functions to simulate the potential field in problems with complex morphologies. While the method is general and can be applied to any problem with existing Green's functions, the current work focuses on Poisson problems. In the proposed Green's Discrete Transformation Method (GDTM), the source points used for evaluating the basis functions are created along a virtual boundary outside the domain of interest. The discrete transformation technique introduced in [29] allows for using a larger number of basis functions than the number of boundary collocation points, which can improve accuracy of the method compared to conventional techniques such as the least squares approximation (LSA) [33]. By implementing Green's functions in place of exponential bases used in the EBFs method, the GDTM provides more appropriate basis functions for simulating problems with complex geometries. However, in comparison with the EBFs method, this feature limits the application range of the GDTM to problems for which the Green's functions can be computed analytically. A detailed numerical study is performed to identify the optimal number and location of the GDTM source points that yield the best approximation of a partition of unity while maintaining a reasonable computational cost. We show that the optimal source points evaluated via this approach can also accurately approximate other potential fields over complex domains with arbitrary domain shapes.

The remainder of this article is structured as follows. Section 2 presents the problem formulation together with describing the GDTM algorithm. In Sections 3 and 4, we study the impact of source points and boundary points on the accuracy and convergence rate of the GDTM. Finally, Section 5 presents the application of this method for simulating two Poisson problems with intricate geometries: the conductive heat transfer in a heterogeneous materials system and the ion concentration in an electrolyte solution formed by pitting corrosion.

2. Problem formulation and GDTM algorithm

Consider a two-dimensional open domain Ω with closure $\overline{\Omega}$ and boundaries $\partial\Omega = \Gamma_D \cup \Gamma_N$ corresponding to the Dirichlet and Neumann boundaries, respectively ($\Gamma_D \cap \Gamma_N = \emptyset$). The strong form of the governing equations for the steady state diffusion of a potential field $u(\mathbf{x})$ (Poisson equation) is expressed as: find u such that

$$\begin{aligned} D\nabla^2 u(\mathbf{x}) &= 0 \quad \text{in } \overline{\Omega} \\ u(\mathbf{x}) &= \bar{u} \quad \text{on } \Gamma_D \\ -\kappa\nabla u(\mathbf{x})\cdot\mathbf{n}(\mathbf{x}) &= \bar{q} \quad \text{on } \Gamma_N, \end{aligned} \tag{1}$$

where D is the diffusivity, κ is a constant coefficient (e.g., thermal conductivity for heat transfer problems), $\mathbf{n}$ is the unit normal vector to the boundary, $\bar{u}$ is the fixed value of the species along Γ_D, and $\bar{q}$ is the applied flux along Γ_N. Assuming a