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Godunov-type schemes with an inertia term for unsteady full Mach number range flow calculations

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ABSTRACT

An inertia term is introduced in the AUSM^{+}-up scheme. The resulting scheme, called AUSM-IT (IT for *Inertia Term*), is designed as an extension of the AUSM^{+}-up scheme allowing for full Mach number range calculations of unsteady flows including acoustic features. In line with the continuous asymptotic analysis, the AUSM-IT scheme satisfies the conservation of the discrete linear acoustic energy at first order in the low Mach number limit. Its capability to properly handle low Mach number unsteady flows, that may include acoustic waves or discontinuities, is numerically illustrated. The approach for building the AUSM-IT scheme from the AUSM^{+}-up scheme is applicable to any other Godunov-type scheme.

1. Introduction

Convective and acoustic waves may propagate together in compressible flows, at time and space scales that may be very different, and with possible interactions. Design of numerical methods able to handle properly these phenomena remains a challenging task, even if viscous effects are neglected. With a co-located arrangement of the unknowns, accuracy and robustness of the numerical method depend on the way of interpolation on the cell or element faces. Two broad categories of methods can be identified, according to the equations they are derived from: (1) Methods solving a Riemann problem at each face by using characteristic equations (these methods are referred to as Godunov-type schemes in the present study); (2) Momentum interpolation methods, derived from the momentum equation. In our opinion, the relations between these two approaches merit investigation in order to improve their respective capabilities.

The difficulties arising at low Mach number when Godunov-type schemes are used have been widely studied, mainly for steady calculations (see *e.g.* [1–3,11]). Denoting by $\mathrm{M_r}$ a reference Mach number in the flow, it has been recognized that avoiding the checkerboard decoupling problem needs a $1/\mathrm{M_r^2}$-scaling of the pressure gradient term in the face velocity or the face mass flux. This $1/\mathrm{M_r^2}$-scaling implies that the thermodynamic and the acoustic pressures are constant in space at the convective scale, which conforms to the continuous asymptotic analysis, provided that suitable boundary conditions are adopted (see *e.g.* [3,6]). For AUSM-type schemes, it was shown by Dellacherie [1] that the $1/\mathrm{M_r^2}$-scaling is also necessary for avoiding spurious acoustic waves when starting from so-called well-prepared initial conditions. However, there

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is experimental evidence that this property does not guarantee accurate calculation of acoustic propagation in low Mach number flows. In our earlier work [15], it was thus observed that for unsteady calculations of low Mach number flows, the presence of the time-step in the pressure–velocity coupling coefficient of the face velocity, as for momentum interpolation, is beneficial. Based on this observation, an improvement of the AUSM$^+$-up scheme was then proposed in [14], by mimicking the pressure–velocity coupling of the momentum interpolation. However, we noted that the quality of the momentum interpolation, if properly defined for unsteady calculations in a Rhie–Chow-like manner (see [15–17]), was not reached for some tests at low Mach number. Improvement of predictions for unsteady low Mach number flows by the AUSM$^+$-up scheme and the related SLAU scheme (Simple Low Dissipative AUSM) by introduction of Strouhal number dependence in the coefficient of the pressure dissipation term in the mass flux expression was also obtained by Sachdev et al. [19]. These authors demonstrated that the scaling of the coupling coefficient has to be quite different for steady low Mach number problems and for unsteady low Mach number problems. By changing the scaling, they proved significant improvement with the AUSM$^+$-up scheme for unsteady low Mach number problems with hydrodynamic coupling between velocity and pressure (vortex propagation) and with acoustic coupling (propagation of a pressure oscillation and propagation of weak shocks and expansion fans). They also illustrated oscillatory behavior of the SLAU method for steady low Mach number flows. However, it remains unclear how to combine the different scaling factors and, for instance, to ensure that the correct steady scaling is obtained for the solution of a steady low Mach number problem calculated with an unsteady method. A similar remark holds for our own work [14]. A particular problem with the AUSM$^+$-up method is that the damping by the pressure difference term in the mass flux expression which is appropriate for steady low Mach number flow is too high for propagation of smooth acoustic signals in unsteady low Mach number flows. On the other hand, as observed by Sachdev et al. [19], the dissipation is too low for propagation of acoustic discontinuities (low Mach number Riemann problems). So, it becomes very delicate to tune the pressure dissipation such that it functions properly for the different types of low Mach number flows. Too low pressure dissipation in the mass flux definition of the SLAU method for simulation of propagation of discontinuities in low Mach number flows was also remarked by Shima [20]. He proved that it is possible to eliminate oscillations by increasing the coefficient of the pressure dissipation term in the mass flux definition by a sensor for wiggles. Of course, the larger dissipation smears somewhat the discontinuities. The conclusion is that methods that rely on tuning of the coefficient of the pressure dissipation term in the mass flux definition in AUSM-type discretizations are very delicate and certainly have not reached maturity yet.

Observing that an inertia term is present in the face velocity expression by the momentum interpolation, and missing in the face velocity expression by the scheme proposed in [14], we propose in the present study to introduce this inertia term in the face velocity of Godunov-type schemes. The momentum interpolation is used as a guideline to accommodate this term. The resulting pressure–velocity coupling exhibits then the suitable $1/\mathrm{M}_\mathrm{r}^2$-scaling for low Mach number steady calculations. In the unsteady case, the pressure–velocity coupling exhibits also the proper Mach number scaling and time-step dependence, identified in [14,15]. Moreover, the inertia term is introduced such that the steady state, if it exists, does not depend on the time-step.

As pointed out in [1,2], an asymptotic property providing insights for the design of Godunov-type schemes that remain accurate at low Mach number is the linear acoustic energy conservation in the low Mach number regime, which holds if periodic boundary conditions are adopted. This property is used in the present study as a guideline to assess the proper way of inertia term interpolation, as well as the proper way of pressure interpolation, in order to enforce the acoustic energy conservation at the discrete level.

The key point is that, if the acoustic component of the pressure is centrally interpolated in the low Mach number limit, the presence of the inertia term in the face velocity enforces acoustic energy conservation at the discrete level. More precisely, the $1/\mathrm{M}_\mathrm{r}$-scaling of the numerical dissipation that arises from the spatial discretization of the linear acoustic wave equation, is thus counterbalanced. Conservation of acoustic energy is clearly a prerequisite for accurate calculation of unsteady low Mach number flows including acoustic features.

2. Foundation of Godunov-type schemes on characteristic equations

In this section, the Mach number scaling of the pressure gradient term in the face velocity expression of Godunov-type schemes is examined in the light of the characteristic equations from which these schemes are drawn.

Reference pressure p_r, density ϱ_r and velocity v_r thought of as a convective quantity, are introduced. A reference Mach number is then defined as $\mathrm{M}_\mathrm{r} = v_\mathrm{r}/\sqrt{p_\mathrm{r}/\varrho_\mathrm{r}}$. Reference length l_r and duration t_r, thought of as a convective quantity, are also considered, as well as a reference Strouhal number, $\mathrm{St}_\mathrm{r} = (l_\mathrm{r}/v_\mathrm{r})/t_\mathrm{r}$. Notice that it is possible to choose the reference length l_r as $t_\mathrm{r}\sqrt{p_\mathrm{r}/\varrho_\mathrm{r}}$, which is an acoustic length. Then, the reference Strouhal and Mach numbers are related by $\mathrm{St}_\mathrm{r} = 1/\mathrm{M}_\mathrm{r}$. Here however, the possibility is left open for another choice of reference duration, so that we will work with the reference Strouhal number St_r. Associated with the Euler equations in dimensional form,

$$\partial_t \varrho + \nabla\cdot(\varrho \boldsymbol{v}) = 0, \tag{1a}$$

$$\partial_t(\varrho \boldsymbol{v}) + \nabla\cdot(\varrho \boldsymbol{v}\otimes\boldsymbol{v}) + \nabla p = 0, \tag{1b}$$

$$\partial_t(\varrho E) + \nabla\cdot(\varrho H \boldsymbol{v}) = 0, \tag{1c}$$

$$E = e + \frac{1}{2}\|\boldsymbol{v}\|^2, \tag{1d}$$