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A Godunov-like point-centered essentially Lagrangian hydrodynamic approach

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ABSTRACT

We present an essentially Lagrangian hydrodynamic scheme suitable for modeling complex compressible flows on tetrahedron meshes. The scheme reduces to a purely Lagrangian approach when the flow is linear or if the mesh size is equal to zero; as a result, we use the term essentially Lagrangian for the proposed approach. The motivation for developing a hydrodynamic method for tetrahedron meshes is because tetrahedron meshes have some advantages over other mesh topologies. Notable advantages include reduced complexity in generating conformal meshes, reduced complexity in mesh reconnection, and preserving tetrahedron cells with automatic mesh refinement. A challenge, however, is tetrahedron meshes do not correctly deform with a lower order (*i.e.* piecewise constant) staggered-grid hydrodynamic scheme (SGH) or with a cell-centered hydrodynamic (CCH) scheme. The SGH and CCH approaches calculate the strain via the tetrahedron, which can cause artificial stiffness on large deformation problems. To resolve the stiffness problem, we adopt the point-centered hydrodynamic approach (PCH) and calculate the evolution of the flow via an integration path around the node. The PCH approach stores the conserved variables (mass, momentum, and total energy) at the node. The evolution equations for momentum and total energy are discretized using an edge-based finite element (FE) approach with linear basis functions. A multidirectional Riemann-like problem is introduced at the center of the tetrahedron to account for discontinuities in the flow such as a shock. Conservation is enforced at each tetrahedron center. The multidimensional Riemann-like problem used here is based on Lagrangian CCH work [8,19,37,38,44] and recent Lagrangian SGH work [33–35,39,45]. In addition, an approximate 1D Riemann problem is solved on each face of the nodal control volume to advect mass, momentum, and total energy. The 1D Riemann problem produces fluxes [18] that remove a volume error in the PCH discretization. A 2-stage Runge–Kutta method is used to evolve the solution in time. The details of the new hydrodynamic scheme are discussed; likewise, results from numerical test problems are presented.

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1. Introduction

The Lagrangian hydrodynamic approach is widely used to calculate problems involving shocks. Lagrangian calculations typically use polyhedral meshes with 8 or more nodes, such as hexahedron, on 3-dimensional (3D) problems. Polyhedral meshes with 8 or more nodes have merits, but they have some weakness in comparison to tetrahedron meshes, which have 4 nodes. Several notable advantages of tetrahedral meshes include: reduced complexity in generating conformal meshes, reduced complexity in mesh reconnection, and preserving tetrahedron cells with automatic mesh refinement. Due to these advantages, we seek to develop an accurate hydrodynamic algorithm suitable for shocks and smooth flows on tetrahedron meshes.

Two common Lagrangian methods are the staggered-grid hydrodynamic approach (SGH) [9,11,12,58,63] and the cell-centered hydrodynamic (CCH) approach [1,2,8,19,37,38,44]. The SGH approach solves the momentum conservation equation and the internal energy evolution equation on staggered control volumes. The control volume for the internal energy evolution equation coincides with the cell. The control volume for the momentum conservation equation encircles the node and is commonly termed the "dual grid". The vertices of the dual grid coincide with the cell centers. In SGH, the kinematic variables such as velocity are stored at the node (*i.e* vertices of the cell) and the thermodynamic variables such as pressure and internal energy are stored at the cell center. The strain is calculated on the cell boundary. The CCH approach is a spatially collocated approach so the conservation equations for momentum and total energy are solved on a single control volume that coincides with the cell boundary. The strain is also calculated on the cell boundary. In this work, we seek to model shocks on tetrahedron meshes, and unfortunately, the SGH and CCH approaches do not necessarily perform well on tetrahedron meshes. Scovazzi [52] performed analysis on the compatible SGH approach [9,11] and showed the approach has undesirable error modes. Scovazzi presented calculations using tetrahedron meshes that support the numerical analysis and the results illustrate the compatible SGH approach does not perform well on tetrahedron meshes. A similar study with triangular grids was performed earlier by Loubère et al. [32]. Loubère demonstrated that the compatible SGH approach does not perform well on triangular grids.

Another Lagrangian approach is the point-centered hydrodynamic (PCH) method [14–16,27,28,51–53]. The PCH approach is a spatially collocated method where the conservation equations for momentum and total energy are solved on the dual grid around the node. Likewise, the strain is calculated on the same dual grid. The PCH approach is of interest to this work because it has been used on triangular and tetrahedron meshes. Furthermore, Scovazzi [52] presents theory and demonstrates that the PCH approach is superior to the compatible SGH [9,11,12] approach on tetrahedron meshes. The Lagrangian PCH approach was actively studied in the context of the Free-Lagrange framework [15]. The concept behind Free-Lagrange is to reconnect the mesh when the mesh becomes deformed. Of interest to this work is the Lagrangian scheme and not the reconnection step. Crowley [15] developed a finite volume PCH approach for incompressible flows on triangular grids. Fritts and Boris [27] followed the work in [15] and proposed a finite difference PCH approach for incompressible free surface flows. Multiple PCH algorithms were developed in the 1980s for compressible flows in the Free-Lagrangian framework. Examples include: the PCH approach by Crowley [16], the PCH approach by Clark [14] in the HOBO code, the PCH approach by Gittings [28] in the TRIX code, and the PCH approach by Sahota [53]. The approach in [28] is of importance because the method replaced the artificial viscosity with a Godunov scheme [29,30]. A different Godunov method was proposed by Addessio et al. [2]. The approach in [2] used a CCH Godunov hydrodynamic scheme [1] on the dual grid, which created a PCH-like approach [25]. The approach by Addessio et al. evolves the dual grid using the Riemann velocities, whereas, the PCH approach evolves the dual grid using the nodal velocities. This distinction is important to this work because an undesirable volume error can arise if the dual grid is evolved using the nodal velocities. The algorithms above used either a finite difference or finite volume discretization. The finite element (FE) approach is a viable alternative to the finite difference and finite volume approaches. Recently, Scovazzi et al. [51,52] proposed a variational finite element PCH approach. The approach was applied to hexahedron and tetrahedron grids. The approach in [51,52] relies on an artificial viscosity model. An alternative FE approach is the edge-based FE Godunov approach. An ALE edge-based FE Godunov PCH approach was proposed by Waltz et al. [61] for tetrahedron grids. In this work, we build on the research in [59–61] and propose an essentially Lagrangian FE PCH Godunov-like method.

The proposed Lagrangian FE PCH approach discretizes the conservation equations for momentum and total energy with linear basis functions. The FE approach used here has many similarities to the finite volume approach. A unique feature of this work is that a multidirectional Riemann-like problem is introduced at the center of the tetrahedron to account for discontinuities in the flow such as a shock. Using a multidirectional Riemann-like solver differs from the edge-based FE PCH Godunov methods which solve an approximate 1D Riemann problem on the edges [59–61]. Conservation is enforced at each tetrahedron center instead of along an edge. The multidimensional Riemann-like problem used here is based on Lagrangian CCH work [8,19,37,38,44] and recent Lagrangian SGH work [33–35,39,45]. The dual grid control volume will evolve as a function of the nodal velocities which introduces a volume error. The volume error is removed by solving an additional 1D Riemann problem on each face of the nodal control volume. The 1D Riemann problem produces mass, momentum, and total energy fluxes [18] that remove the volume error. A 2-stage Runge–Kutta method is used to evolve the solution in time.

The layout of the paper is as follows. The nomenclature used in the paper is discussed in Section 2. The governing equations are discussed in Section 3. The multidirectional Riemann-like problem at the tetrahedron center is discussed in Section 4. The volume discretization, associated volume change error, and volume correction is discussed in Section 5. The 1D Riemann corrective fluxes are discussed in Section 6. The details on extending the algorithm to 2nd order is discussed