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# Local time–space mesh refinement for simulation of elastic wave propagation in multi-scale media

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This paper presents an original approach to local time–space grid refinement for the numerical simulation of wave propagation in models with localized clusters of micro-heterogeneities. The main features of the algorithm are

– the application of temporal and spatial refinement on two different surfaces;
– the use of the embedded-stencil technique for the refinement of grid step with respect to time;
– the use of the Fast Fourier Transform (FFT)-based interpolation to couple variables for spatial mesh refinement.

The latter makes it possible to perform filtration of high spatial frequencies, which provides stability in the proposed finite-difference schemes.
In the present work, the technique is implemented for the finite-difference simulation of seismic wave propagation and the interaction of such waves with fluid-filled fractures and cavities of carbonate reservoirs. However, this approach is easy to adapt and/or combine with other numerical techniques, such as finite elements, discontinuous Galerkin method, or finite volumes used for approximation of various types of linear and nonlinear hyperbolic equations.



## 1. Introduction

The numerical simulation of seismic wave propagation is one of the main tools used in seismic prospecting and seismology. Full-scale simulation can be used to study the peculiarities of wave propagation in complex models, such as anisotropic [1,2], viscoelastic [1], and poroelastic models [3,4], and models with irregular topography [5–7], etc. Moreover, numerical simulation is an essential element in seismic imaging procedures such as Reverse Time Migration and Full Waveform Inversion [8]. Currently, one of the most active developing directions in seismic processing is the imaging of scattered waves to gain detailed knowledge about the internal structure of fractured/cavernous reservoirs, including a scattering

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index to predict the orientation of fractures [9], the scattering energy to locate cavernous/fractured reservoirs [10,11] and the coda wave energy [12]. However, the implementation of the named techniques and the study of physical aspects of wave propagation in models with a small-scale structure require numerical methods that are able to match the peculiarities of the model, which can be as small as 0.01 of the wavelength. A simple estimation shows that for a typical model of the size $100 \times 100 \times 30$ wavelengths, the use of a mesh matching the small-scale heterogeneities (2–5 points per obstacle) results in a grid with $2.5 \times 10^{12}$ points. In the simple case of isotropic elastic media, one may use the standard staggered grid scheme (SSGS) [13]; thus, 12 variables per grid node are to be stored and approximately 104 Tb of RAM is needed for the simulation. Similar considerations make it possible to estimate the flops needed to simulate the wavefield of a single shot of up to 100 wave periods, using the number of 0.5 Eflop. However, a regular 3D seismic survey consists of thousands of source gathers, which makes the numerical simulation of a real acquisition for a model with small-scale heterogeneities impossible on modern supercomputers.

It is worth mentioning two main approaches to this statement.

1. The use of homogenization techniques leading to the so-called "effective media". These include such approaches as the use of Biot poroelastic media [14]; homogenization of fine-layered media [15,16]; different techniques to upscale fractured media: first and second order Hudson theory [17,18]; the noninteracting crack theory [19]; differential and extended differential approaches [20,21]; and the T-matrix method [22].
   Based on the recent theoretical developments, it appears possible to create effective models for media with micro-heterogeneities of rather complicated structures such as cracks of different orientations, families of cracks, cracks in prestressed media [23–25]. However, these techniques have serious drawbacks. Above all, this brings about *homogeneous* effective media and, therefore, cancels scattered waves possessing significant information about the location and properties of microstructures. Moreover, all upscaling approaches are constructed under the assumption of a static nature of the process, that is, they are precise as the size of heterogeneities tends to zero with respect to the dominant wavelength [26]. Thus, these types of approaches can be considered to be asymptotic methods.
2. We should also mention advanced averaging techniques such as those presented in [27–29]; multi-scale finite elements and multi-scale discontinuous Galerkin methods [30–32]; and numerical averaging within a given frequency range [33]. These approaches were designed primarily for the numerical simulation of wave propagation, preserving more information of the model microstructure than the previously mentioned techniques; however, it is rather unclear which physical features of the model are reflected in the numerical solution obtained by these techniques. So, these approaches can be considered as an intermediate link between upscaling and direct multi-scale modeling.

The simulation of seismic scattering/diffraction for cavernous/fractured reservoirs is a mesoscale problem. In fact, on the one hand, asymptotic methods are unapplicable because they cancel the scattering energy, which conveys important and useful information about the fine structure of reservoirs. On the other hand, the size of obstacles is too small for a straightforward application of conventional numerical techniques.

In this paper, we introduce an approach to the numerical simulation of wave propagation in media with subseismic-scale heterogeneities such as cavities and fractures, based on local mesh refinement with respect to time and space. One of the main features of the particular problem is that obstacles are concentrated inside some relatively small volume (layer), and there is no need to describe extremely precisely the shape of each single obstacle but rather their distribution within a given volume. Note that typically, the models of fractured reservoirs are derived by means of a statistical simulation [34], and they are provided on some sufficiently fine regular grid [35]. So, we admit that the Galerkin-type approaches on irregular adaptive meshes are suitable for the simulation of wave propagation with a few small-scale obstacles [36,37]. However, for complex models the finite differences are still preferred due to a combination of simplicity, universality, and suitable accuracy [8,38]. Thus, in this paper, we focus on the finite differences with a local discontinuous grid refinement. As explicit time stepping is used, the time step is defined by the smallest spatial step that reduces the performance of the algorithm if there is local refinement in space. This leads to the necessity to refine both the spatial and temporal meshes.

Several approaches to the implementation of the local time–space mesh refinement were proposed in recent decades. The simplest among them is based on the interpolation of the solution with respect to time at the refinement interface [39]. However, as shown in [40], this approach may cause problems with stability. Another set of approaches is based on solving the generalized Riemann problem to perform time stepping. See [41–44] for a finite volume method and [45,46] for a discontinuous Galerkin method. A completely different technique was proposed in [47], studied in [48,49] and developed in [50] and [51], where the conditions at the interface are constructed to conserve the total energy of the solution, thus ensuring stability. Another technique has been recently reported in [52–54], where the connection between the grids is realized by means of equivalent currents. However, the mentioned techniques have two general drawbacks. First of all, they are too complicated to be applied to realistic 3D models, and to our knowledge, 3D results have only been presented in [43,45,46,54,55]. Second, regardless of the type of mesh refinement, it always possesses artificial reflections from the interface where two grids are coupled. These reflections occur because of the numerical dispersion and the way solutions are connected at the interface. Many researchers were studying the convergence of their methods and have proved it is of a needed order. Thus, artificial reflections converge to zero with at least the same order. However, this may not be sufficient in practice, where a low level of artificial reflections is required for sufficiently rough grids, that is, with 10 to 30 points per wavelength for a coarse mesh. Some of these algorithms possess reflections as high as 0.1 in amplitude of