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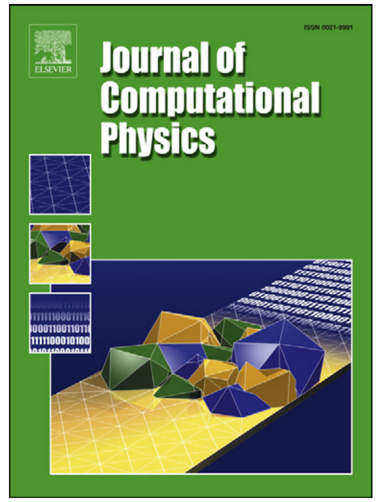

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Zhao-peng Hao, Zhi-zhong Sun, Wan-rong Cao

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A fourth-order approximation of fractional derivatives with its applications *

Zhao-peng Hao† Zhi-zhong Sun‡ Wan-rong Cao§

Department of Mathematics, Southeast University, Nanjing 210096, P.R. China

Abstract

A new fourth-order difference approximation is derived for the space fractional derivatives by using the weighted average of the shifted Grünwald formulae combining the compact technique. The properties of proposed fractional difference quotient operator are presented and proved. Then the new approximation formula is applied to solve the space fractional diffusion equations. By the energy method, the proposed quasi-compact difference scheme is proved to be unconditionally stable and convergent in $L2$ norm for both 1D and 2D cases. Several numerical examples are given to confirm the theoretical results.

Keywords: high-order approximation, fractional derivative, fractional differential equation, quasi-compact difference scheme

1 Introduction

During the past decades, fractional calculus has been playing more and more important roles in many fields. The space fractional diffusion equation is one of the popular mathematical models of fluid flow in porous materials, anomalous diffusion transport, chemistry, etc.(see [11, 20, 23, 28]). Though there are some analytical methods like the Fourier transform method, the Laplace transform method, the Mellin transform method, and the Green function method, it is still difficult to evaluate the fractional derivative for most functions, and the exact solution of the fractional diffusion equation is hardly to be given either. Hence, it is essential to develop the efficient numerical methods.

Due to the equivalence of the Riemann-Liouville (RL) derivative and Grünwald Letnikov (GL) derivative under smooth assumptions imposed on the initial value [23], the GL definition is commonly used to approximate the RL derivative with first order. However, when applying the approximation to the space fractional partial differential equation, it leads to an unstable scheme. To overcome this difficulty, an integer shifted (right shift for left side derivative or left shift for right derivative) Grünwald formula was proposed by Meerschaert and Tadjeran [15] to approximate the space fractional derivative, and successfully applied to solve the advection-dispersion equations. Based on the shifted Grünwald formula, Meerschaert et al. did a series of work for numerically solving the space fractional diffusion equations [14, 16, 34, 35].

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†E-mail address: haochpeng@126.com

‡Corresponding author; E-mail address: zzsun@seu.edu.cn

§E-mail address: wanrongcao@gmail.com