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An explicit residual based approach for shallow water flows

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We describe fully explicit residual based discretizations of the shallow water equations with friction on unstructured grids. The schemes are obtained by properly adapting the explicit construction proposed in Ricchiuto and Abgrall (2010) [57]. In particular, previous work on well balanced integration (Ricchiuto, 2011 [56]) and preservation of the depth non-negativity (Ricchiuto and Bollermann, 2009 [60]) is reformulated in the context of a genuinely explicit time stepping still based on a weighted residual approximation. The paper discusses in depth how to achieve in this context an exact preservation of all the simple known steady equilibria, and how to obtain a super-consistent approximation for smooth non-trivial moving equilibria. The treatment of the wetting/drying interface is also discussed, giving formal conditions for the preservation of the non-negativity of the depth for a particular case, based on a nonlinear variant of a Lax–Friedrichs type scheme. The approach is analyzed and tested thoroughly. The quality of the numerical results shows the interest in the proposed approach over previously proposed schemes, in terms of accuracy and efficiency.

1. Introduction

Free surface flows are relevant in a large number of applications, especially in civil and coastal engineering. The problems concerned are either (relatively) local, such as dam breaks and flooding, overland flows due to rainfall, nearshore wave propagation and interaction with complex bathymetries/structures, and tidal waves in rivers, or global such as in ocean or sea basin models for the study of e.g. tsunami generation and propagation.

The simulation of such flows can be carried out by solving directly the three dimensional Navier–Stokes equations. However, for many applications, including e.g. nearshore wave propagation and flooding, simplified models obtained by combining vertical averaging and some form of thin layer approximation provide reliable results. The applicability of such models depends on the nature of the flow and on the hypotheses at their basis [15,44].

The simplest among these models is the so-called shallow water model. The model assumes that the waves developing in the flow are *long* (small ratio amplitude/wavelength), and of a hydrostatic vertical variation of the pressure [35,48]. More complex nonlinear models can be obtained, by including higher order terms, and depending on the hypotheses on the flow [35,48,44,15]. The first order shallow water approximation constitutes a non-homogeneous hyperbolic system where the effects of the variation of the bathymetry and the viscous friction on the bottom are modeled by the source terms [35,48].

The amount of literature related to the solution of the shallow water system is vast. This model finds applications in oceanography, hydrology, and meteorology (see e.g. [20,34,40,72–74] and references therein). The main challenges when solving the shallow water system numerically are related to the discretization of the bathymetry and friction terms, and to the numerical treatment of nearly dry regions. For the first issue, one speaks often *asymptotic preserving* character or *well balancedness* of a discretization. The second issue is what is referred to as the wetting/drying strategy.

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Well balancing, refers to the ability of the discretization to preserve some steady equilibria involving the existence of a set of invariants exactly, or within some mesh size dependent bounds possibly more favorable than the accuracy of the scheme. The typical example is the so-called *lake at rest state* involving a flat still free surface, that should remain flat whatever the shape of the bottom. This property is what one refers to as *Conservation property, or C-property* [14] or well-balancedness [36]. One speaks of approximate C-property when the steady state is kept within an accuracy higher than that of the underlying scheme. This property becomes important when one is interested in flows that, at least locally, are perturbations of one of these steady equilibria, so that numerical perturbations might interfere with the actual flow giving wrong results. There is plenty of literature discussing several different approaches to the preservation of steady equilibria, in particular the so-called lake at rest state. Most of these developments have taken place in the finite volume community, and are thought in terms of one-dimensional flows (see e.g. [14,33,36,51] and references therein). The basic approach boils down either to the inclusion of a source term contribution in the FV numerical flux, so that the correct equilibrium is found at the discrete level [14,36,41], or to the rewriting of the system in a relaxation form, where an appropriate integral of the source term is added to the physical flux in the Maxwellian on the right hand side [29,70]. The multidimensional case is often handled by a dimension by dimension extension on structured grids (see [50–52,80], for recent examples), or by introducing local pseudo-one dimensional problems along some geometrical directions (e.g. normals to grid faces) [12,28, 41,49]. These modified FV fluxes are also used in the context of discontinuous Galerkin schemes to retain the C-property (see e.g. [32,78]). A different approach is that of the well balanced wave propagation finite volume schemes of LeVeque and his co-workers [45,46], the continuous stabilized finite element discretizations proposed by G. Hauke [38], and residual distribution schemes of [17,58,60].

On the other hand, the computational treatment of nearly dry areas involves the solution of the following issues: ensuring that in these regions no unphysical negative depths are obtained; handling some ill-posed problems such as the computation of the local velocity given depth and discharge; preserving the well balanced character of the method when $0 < H \ll 1$.

These three issues are not independent and the large majority of the wetting/drying treatments discussed in literature boil down to: rely on the use of some positivity preserving scheme to be able to keep the depth non-negative; introducing a cut-off of some sort on the velocity (and mass flux) to avoid zero over zero type divisions; modify the *numerical* slope of the bathymetry used in the discrete equations; employ an implicit (split or unsplit) treatment of the friction term to handle the stiffness associated to this term in dry areas. These ideas can be put in practice in various ways, depending on the initial formulation of the method, on the techniques used to reach higher order of accuracy, and on the type of nonlinear mechanism used to combine high order and preservation of the positivity. For an overview see [12,18,19,21,22,32,49,81,79].

This paper follows the author's previous work [56,58,60,59] on the construction of residual approximations to the shallow water system. The main objective is to propose a method more efficient than those proposed in the last references, yet retaining all the nice properties of these methods. These properties include the C-property and a generalized C-property for constant energy flows, the preservation of the depth non-negativity, and a robust treatment of moving shorelines. All these properties are achieved on general adaptive unstructured grids. This has a definite advantage as it allows an enhanced resolution of local features, such as e.g. steep variations of the bathymetry leading to complex local flow patterns. More advanced enhancements can be obtained by means of dynamic adaptation for time dependent flows, but this is not considered in this paper. The major limitation of the schemes of [56,58,60,59] is that, while being genuinely implicit and highly nonlinear, they still need to satisfy an explicit type constraint on the allowed time step for the preservation of the depth's non-negativity. Possible routes to overcome this limitation have been suggested in [42,68], using an unconditionally monotonicity preserving space–time framework, and in [57], where a fully explicit variant of the residual distribution method is proposed. In this paper, we develop the ideas of the last reference, and propose a specific formulation for the shallow water equations.

In particular, the main contributions of the present work are: a detailed analysis of the conditions leading to the respect of the C-property for both lake at rest solutions and flows over constant slopes with friction; a formal characterization of the approximate generalized C-property as referred in [56] (C-property for constant energy flows), here studied in terms of super-consistency with the steady solution both on general grids, and on flow aligned cartesian meshes; the analysis of the preservation of the non-negativity of the depth for the explicit schemes based on the nonlinear Lax–Friedrichs method of [57]; an extensive validation and comparison with the implicit scheme of [60]. Few of these results have been presented in [55] (see also the manuscript [54]).

The paper is organized as follows. We recall the form of the shallow system and a number of exact steady equilibria in Section 2. The explicit residual discretization approach is then recalled in Section 3. Section 4 finally analyzes the properties of the discretization, namely well balancedness (C-properties), accuracy, positivity preservation, and wetting/drying strategy. Lastly, in Section 5 we demonstrate the capabilities of the scheme on a large number of numerical tests. Conclusive remarks and future developments end the paper in Section 6.

2. The shallow water equations

The system of the Nonlinear Shallow Water Equations (NLSW) reads

$$\partial_t H + \nabla \cdot (H\vec{v}) + R(x, y, t) = 0$$