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Scale separation for multi-scale modeling of free-surface and two-phase flows with the conservative sharp interface method

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ABSTRACT

In this paper we present a scale separation approach for multi-scale modeling of free-surface and two-phase flows with complex interface evolution. By performing a stimulus-response operation on the level-set function representing the interface, separation of resolvable and non-resolvable interface scales is achieved efficiently. Uniform positive and negative shifts of the level-set function are used to determine non-resolvable interface structures. Non-resolved interface structures are separated from the resolved ones and can be treated by a mixing model or a Lagrangian-particle model in order to preserve mass. Resolved interface structures are treated by the conservative sharp-interface model. Since the proposed scale separation approach does not rely on topological information, unlike in previous work, it can be implemented in a straightforward fashion into a given level set based interface model. A number of two- and three-dimensional numerical tests demonstrate that the proposed method is able to cope with complex interface variations accurately and significantly increases robustness against underresolved interface structures.

1. Introduction

Two-phase flows and flows with interfaces constitute an area of intense research and strong practical and industrial impact [1]. Phenomena such as drop impact [2,3] and atomization processes [4,5] require handling of a wide range of physical scales in numerical simulation, which is a formidable challenge [6]. Several efficient numerical methods have been developed and implemented successfully to tackle multi-scale problems, such as adaptive mesh refinement (AMR) [1,7–9] and multi-resolution (MR) [10–15] methods. As methods using an explicit representation of the interface such as arbitrary Lagrangian–Eulerian (ALE) discretizations [16] or front-tracking (FT) methods [17] encounter problems for complex interface evolution [1], methods using an implicit representation of the interface such as VOF [18] or level set [19–21] have been widely employed for interfacial flows [1,5,22,23].

With sharp-interface methods (SIM) [17,22,24], accurate modeling of interface interactions is essential. The conservative sharp-interface method [22,25] approximates the segment of an interface within a computational cell as planar. Interfacial scales of magnitude δ can be classified as resolved if they are much larger than the grid size h ($\delta \gg h$), and as non-resolved if $\delta \ll h$. Resolved-scale interface evolution can be computed accurately by linear approximation of the interface segments cut by a computational cell. For given spatial resolution non-resolved scales typically are suppressed by the numerical model as otherwise they may incur numerical instability. Most problematic are marginally resolved scales with $\delta \approx h$. They often

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appear as isolated drops or thin filaments, and are not negligible with respect to the given spatial resolution. A linear approximation of these structures leads to erroneous or even numerically unstable simulations.

As marginally resolved interface scales imply variations of local interface topology, the level set method is well suited for interface tracking. The Refined Level Set Grid (RLSG) method has been introduced by Herrmann [5,23,26], where the level set equations are solved on a separate, high resolution Cartesian grid to cope with interface scales smaller than the mesh size. Although the concept of scale separation was not referred to explicitly in [23], the proposed algorithms for "broken-off drop identification" and "filament transfer" actually serve this purpose. The RLSG method adopts a domain-traversing procedure to identify each contiguous liquid structure. Non-resolved liquid structures are removed by simply changing the sign of their level set values. This method is quite complex and can lead to significant computational overhead. Inspired from RLSG an alternative concept for multi-scale interface modeling of interfacial flows has been explored by Hu et al. [27]. The essential step of this concept is the identification of resolved and non-resolved interface segments by a scale separation scheme. For this purpose the difference between the volume fractions reconstructed on a refined grid and a coarser grid are evaluated. If the difference for a computational cell is larger than a threshold, this cell is considered to contain a non-resolved interface segment. A drawback of such refinement-based methods is that the required refined resolution for the level set could render the simulation prohibitively expensive both in computational time and in memory.

More recently, an adaptive multi-resolution method for multi-phase flows coupled with a sharp interface model has been proposed by Han et al. [28]. A narrow-band level-set technique was applied for interface tracking, reducing both computational overhead and storage. Combined with other speed-up techniques, such as an adaptive multi-resolution scheme (MR) and multi-threading algorithms [29], it provides a suitable tool for simulating complex multi-phase flows on high resolution grids. Nevertheless, as a common feature with previous approaches, marginally resolved interface instabilities can be observed which can create non-resolved interface structures. Although the previously mentioned scale-separation methods can also be applied to such a tool, their embedded-refinement algorithms require considerable effort for parallelization and implementation on heterogeneous computing architectures. The identification and handling of non-resolved structures on a grid with the same resolution as the fluid grid is very desirable for algorithmic and efficiency reasons.

In this paper, a new scale separation approach for interface evolution is developed. Based on the observation that resolved and non-resolved interface segments evolve differently when they are subjected to a small disturbance, separation of these scales can be achieved through a stimulus-response operation during level-set reinitialization. The stimulus-response algorithm is dimension independent and facilitates object-oriented implementation for two and three dimensions. For an assessment of the effectivity of the proposed method for interface scale separation, a number of numerical tests with complex interface evolution are considered.

2. Governing equations

For an inviscid and compressible fluid the governing equations can be written as a system of conservation laws

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_i(\mathbf{U})}{\partial x_i} = 0. \tag{1}$$

Here, $\mathbf{U} = (\rho, \rho u_1, \rho u_2, \rho u_3, E)^T$ is the density of conserved quantities of mass, three components of momentum and total energy with relation $E = \rho e + \frac{1}{2}\rho(u_1^2 + u_2^2 + u_3^2)$, where e is internal energy per unit mass. With a Kronecker delta function δ_{ij}, the corresponding flux vectors $\mathbf{F}_i(\mathbf{U})$ are given by

$$\mathbf{F}_i(\mathbf{U}) = \begin{pmatrix} \rho u_i \\ \rho u_1 u_i + \delta_{i1} p \\ \rho u_2 u_i + \delta_{i2} p \\ \rho u_3 u_i + \delta_{i3} p \\ (E + p) u_i \end{pmatrix}. \tag{2}$$

To close this set of equations it is necessary to specify equations of state (EOS) for each individual phase.

For an ideal gas, the pressure is determined from

$$p = (\gamma - 1)\rho e, \tag{3}$$

where γ is the ratio of specific heats.

For water-like fluids, we apply Tait's equation of state [24]. The temperature is assumed to be constant at 293 K, and pressure p and total energy E are functions of density ρ

$$p = B\left(\frac{\rho}{\rho_0}\right)^{\gamma} - B + A, \tag{4}$$

$$E = \frac{1}{\gamma - 1}(p + B - A) + B - A + \frac{1}{2}\rho\left(u_1^2 + u_2^2 + u_3^2\right). \tag{5}$$

The parameters γ, B, A and ρ_0 are constant. Since the total energy E for water-like fluids is only a function of density and velocity, the energy evolution is omitted from Eqs. (1) and (2).